E-BOOK

FOR POLYTECHNIC STUDENT

INTEGRATION

DBM20023
ENGINEERING MATHEMATICS 2

ANISAH ARBAIN - KAMAL HARON
MOHD TAUFIK SYAZELI

ENGINEERING MATHEMATICS 2

For polytechnics

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POLITEKNIK NILAI
NEGERI SEMBILAN

e ISBN 978-967-2742-05-0

First edition 2021

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Publisher:

Politeknik Nilai Negeri Sembilan (PNS)

Kompleks Pendidikan Bandar Enstek, 71760, Bandar Enstek,

Negeri Sembilan

PREFACE

INTEGRATION- A student's handbook is written as a reference for student enrolled in course DBM20023 - Engineering Mathematics 2 at Polytechnics Malaysia. The e-book contains nine subtopics: Indefinite Integral, Definite Integral, Integrals of Trigonometric Function, Integrals of Reciprocal Function, Integrals of Exponential Function, Integration by Parts, Integration of Partial Fraction and Apply the Techniques of Integration. The notes provided in this book have been written based on syllabus of the course and supported with diagrams for better understanding. On top of that, the practices and assessment provided in this e-book are tailored to suits the needs of students in understanding the topic

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3rd Topic -Integration

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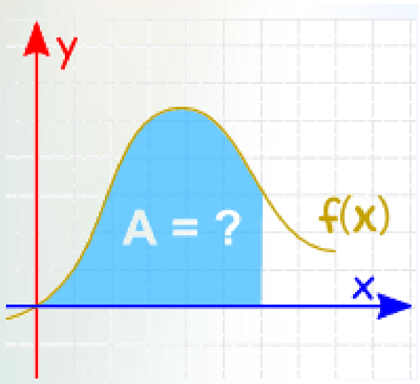
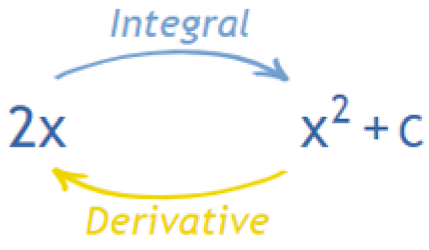
3.0 INTEGRATION

DIFFERENTIATION ↔ INTEGRATION

$$\frac{dy}{dx} \leftrightarrow \int dx$$

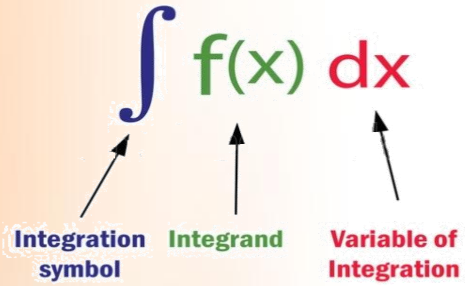


- Integration is the inverse or reverse process of differentiation
- The process of obtaining y from $\frac{dy}{dx}$ is know as integration
- The symbol for the integration is $\int$
- Integration can be used to find areas, volumes, central points and many useful things. It is often used to find the area underneath the graph of a function and the x-axis.
- The first rule to know is that integrals and derivatives are opposites!



Sometimes we can work out an integral
because we know a matching derivative

3.1 INDIFINITE INTEGRAL



- without upper and lower limits,
- also called an antiderivative.

FORMULA

INDEFINITE INTEGRALS	INDEFINITE INTEGRALS
$\int 0\, dx = c \quad ; n \neq 1$	$\int kf(x)\, dx = k \int f(x)\, dx$
$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c\, ; n \neq 1$	$\int ax^n\, dx = \frac{ax^{n+1}}{n+1} + c\, ; n \neq 1$
$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$	$\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \;\; ; n \neq 1$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int cos(x)\, dx = sin\, x + c$
$\int e^x\, dx = e^x + c$	$\int sin(x)\, dx = -cos(x) + c$
$\int e^{ax}\, dx = \frac{e^{ax}}{a} + c$	$\int sec^n(x)\, dx = tan(x) + c$

BASIC INTEGRATION FUNCTION

CONSTANT FUNCTION

$$\int 0\,dx = c$$

$$\int k\,dx = kx + c$$

ALGEBRAIC FUNCTION

$$\int x\,dx = \frac{x^{1+1}}{1+1} + c$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c$$

ADDITION & SUBTRACTION

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

IMPORTANT!!!!

NEED TO CHANGE BEFORE SOLVE THE PROBLEM

1. Square root power

#POWER cannot be '-1'

$$\sqrt[5]{2x+3} \rightarrow (2x+3)^5$$

2. Fraction

$$\frac{5}{x^2} \rightarrow 5x^{-2}$$

3. Expand

$$(2x+3)(1-x) \rightarrow x - 2x^2 + 3$$

$$(3-x)^2 \rightarrow 9 - 6x + x^2$$

4. Separate

$$\frac{3x - 9x^2 + 3}{3x^8} \rightarrow x^{-7} - 3x^{-6} + x^{-8}$$

BASIC INTEGRATION FUNCTION

HOW TO INTEGRATE ?

STEPS:

- **POWER** is added by 1
- Denominator = **NEW POWER**
- + c (indefinite integral)

Step 1

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Step 2

Step 3

INTEGRATION OF ALGEBRAIC FUNCTION

Example | Integrate:

a] $\int 7x \, dx$

$= \frac{7x^{1+1}}{1+1} + c$

$= \frac{7x^2}{2} + c$

b] $\int x^2 \, dx$

$= \frac{x^{2+1}}{2+1} + c$

$= \frac{x^3}{3} + c$

c] $\int 6x^{-2} \, dx$

$= \frac{6x^{-2+1}}{-2+1} + c$

$= \frac{7x^{-1}}{-1} + c$

$= \frac{7}{-x} + c$

BASIC INTEGRATION FUNCTION

INTEGRATION OF CONSTANT

Example | Integrate:

a] $\int 6dx$

$= 6x + c$

b] $\int 0\, dt$

$= c$

c] $\int dm$

$= m + c$

INTEGRATION OF A FUNCTION INVOLVING ADDITION & SUBTRACTION

Example | Integrate:

$a]\ \int (9b^3 + 5)db$

$$= \frac{9b^{3+1}}{3+1} + 5b + c$$

$$= \frac{9b^4}{4} + 5b + c$$

$b]\ \int (x+3)(2x-1)dx$

expand

$$= \int 2x^2 - x + 6x - 3\, dx$$

$$= \int 2x^2 + 5x - 3\, dx$$

$$= \frac{2x^{2+1}}{2+1} + \frac{5x^{1+1}}{1+1} - 3x + c$$

$$= \frac{2x^3}{3} + \frac{5x^2}{2} - 3x + c$$

$c]\ \int \left(\frac{3t^9 + 2t}{t^5}\right) dt$

$$= \int 3t^4 + 2t^{-4} dt$$

$$= \frac{3t^{4+1}}{4+1} + \frac{2t^{-4+1}}{-4+1} + c$$

$$= \frac{3t^5}{5} + \frac{2t^{-3}}{-3} + c$$

$$= \frac{3t^5}{5} - \frac{2}{3t^3} + c$$

BASIC INTEGRATION FUNCTION

INTEGRATION OF CONSTANT

Exercises | Integrate:

a] $\int 12\, dx$

b] $\int 3\pi^2 dt$

c] $\int ds$

d] $\int \frac{3bc}{d} dx$

e] $\int 2.5 \sin x \, dx$

f] $\int \sqrt{65}\, dx$

INTEGRATION OF ALGEBRAIC FUNCTION

Exercises | Integrate:

a] $\int 18x\, dx$

b] $\int 25x^9 dx$

c] $\int \frac{5}{7} x^{-7} dx$

d] $\int \frac{5x^{2/5}}{7}\, dx$

e] $\int \frac{-4}{\sqrt{9x}} dx$

f] $\int \sqrt{\pi t}\, dt$

g] $\int \frac{-1}{\sqrt{4}} dx$

h] $\int \frac{t}{4} dt$

i] $\int 15\sqrt[3]{x}dx$

j] $\int -15m^{-2/3} dm$

k] $\int \left(\frac{2}{3x}\right)^2 dx$

l] $\int 2\sqrt{\pi^2 x^3} dx$

3.1.2 INTEGRATION OF A FUNCTION INVOLVING ADDITION & SUBTRACTION

Exercises | Integrate:

a] $\int (3b^3 - 15b + 7) db$

b] $\int -3(2b^3 + 2b) db$

c] $\int x(x+5) dx$

d] $\int \left(\frac{3b^3}{5} - \frac{2}{b^3} + 7b\right) db$ e] $\int \left(\frac{5s^3}{25} - \frac{2}{4s^8} + \sqrt{s}\right) ds$ f] $\int \left(\frac{4u^3}{7\pi} - \frac{2}{\sqrt{u}}\right) du$

g] $\int (x+5)(3x-1)dx$ h] $\int (2x-1)^2 dx$ i] $\int \frac{(x+5)(3x-1)}{15x^4} dx$

j] $\int \frac{(2x+1)(3x-5)}{\sqrt[3]{x}} dx$ k] $\int \frac{(x+5)^2}{\sqrt[7]{x}} dx$ l] $\int \left(\frac{1}{x^2} - x + \frac{1}{3x^5}\right) dx$

3.2 INTEGRATION OF AN ALGEBRAIC FUNCTION

INTEGRATION OF COMPOSITE FUNCTION

HOW TO INTEGRATE ?

STEPS:

- **POWER is added by 1**
- **denominator =**

  $\left(\frac{dy}{dx}\right.$ **bracket** $\left.\right)$**x (new power)**
- **+ c (indefinite integral)**

FORMULA METHOD

$$\int (ax+b)^n \, dx$$

Power is +1

$$= \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

New Power

$\frac{dy}{dx}$ bracket

INTEGRATION OF ALGEBRAIC FUNCTION

Example | Integrate:

a] $\int (x+4)^4 dx$

Power is +1

$$= \frac{(3x+4)^{4+1}}{3(4+1)} + c$$

$\frac{dy}{dx}$ bracket

New Power

$$= \frac{(3x+4)^5}{15} + c$$

b] $\int \frac{2}{(9x+4)^4} dx$

Change the form

$$\int 2(9x+4)^{-4} dx$$

$$= \frac{2(9x+4)^{-4+1}}{9(-4+1)} + c$$

$$= \frac{2(9x+4)^{-3}}{9(-3)} + c$$

$$= \frac{2}{-27(9x+4)^3} + c$$

c] $\int \sqrt[4]{1-3x} dx$

Change the form

$$\int (1-3x)^{\frac{1}{4}} dx$$

$$= \frac{(1-3x)^{\frac{1}{4}+1}}{-3\left(\frac{1}{4}+1\right)} + c$$

$$= \frac{(1-3x)^{\frac{5}{4}}}{-3\left(\frac{5}{4}\right)} + c$$

$$= \frac{4(1-3x)^{\frac{5}{4}}}{-3(5)} + c$$

$$= \frac{4\sqrt[4]{(1-3x)^5}}{-15} + c$$

Integration of AN ALGEBRAIC FUNCTION

3.2.1 FORMULA METHOD

Exercises | Integrate

:

a] $\int (7x+8)^3\,dx$

b] $\int (3+8x)^7\,dx$

c] $\int (x+15)^{-3}\,dx$

d] $\int 4(3m+9)^3\,dm$

e] $\int \sqrt{(2s-7)^3}\,ds$

f] $\int \sqrt[5]{(3m-19)^3}\,dm$

g] $\int (8-x)^{2/3}\,dx$

h] $\int (2-x)^{1/5}\,dx$

i] $\int \frac{2}{7}(x-5)^{-3}\,dx$

j] $\int \frac{5}{4(3m+2)^3}dm$

k] $\int \frac{8}{\sqrt{(2s-8)^3}}ds$

l] $\int \frac{9}{\sqrt[5]{(1-m)^3}}dm$

Integration of AN ALGEBRAIC FUNCTION

3.2.2 INTEGRATION OF COMPOSITE FUNCTION

SUBSTITUTION METHOD

i. $\int (ax+b)^n \, dx$

ii. $\int f[g(x)] \times g'(x) dx$

iii. $\int \frac{g'(x)}{f[g(x)]} dx$

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$

where $u = g(x)$ $du = g'(x)$

Characteristic Of 'u' :

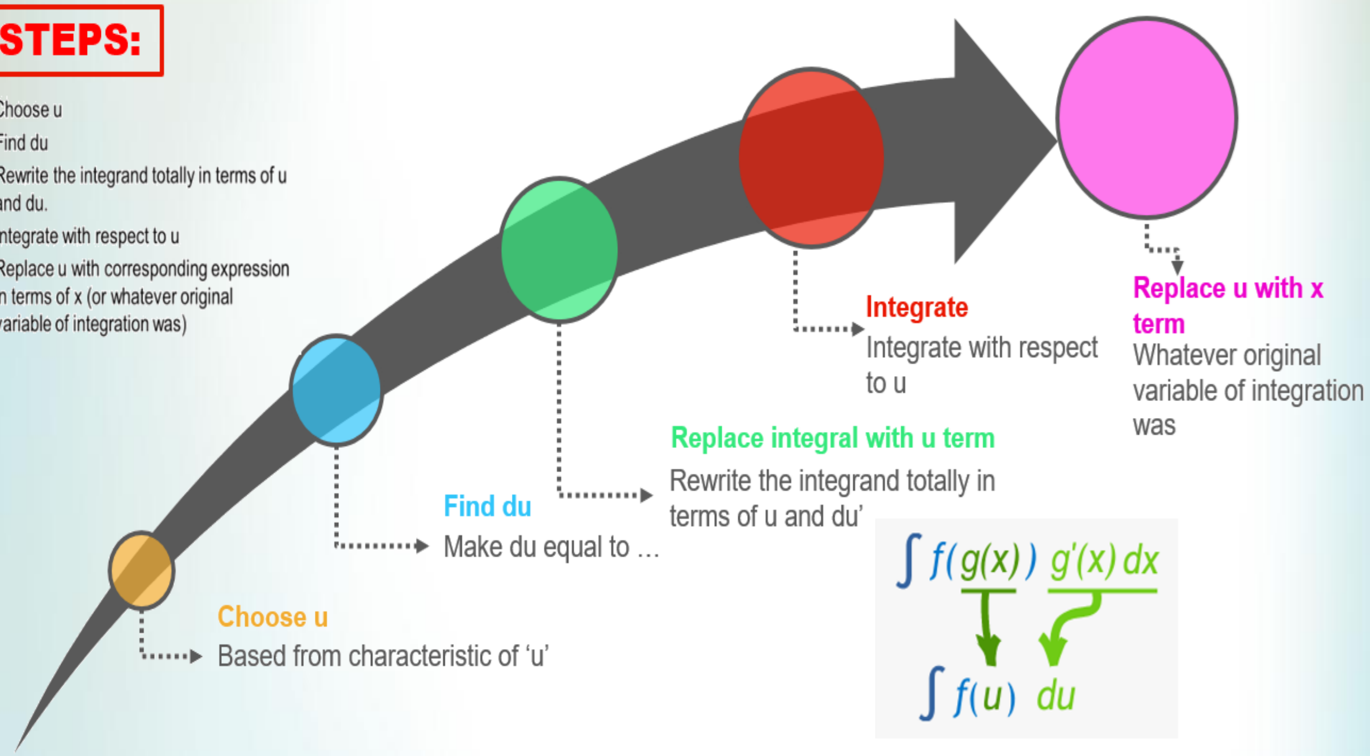
- ❑ **The term under a ROOT**
- ❑ **The HIGHEST POWER**
- ❑ **The term in the DENOMINATOR**
- ❑ **The EXPONENTIAL function**
- ❑ **The RECIPROCAL function**
- ❑ **The term inside a TRIGONOMETRY function**

Integration by AN ALGEBRAIC FUNCTION

HOW TO INTEGRATE SUBSTITUATION METHOD?

STEPS:

1) Choose u
2) Find du
3) Rewrite the integrand totally in terms of u and du.
4) Integrate with respect to u
5) Replace u with corresponding expression in terms of x (or whatever original variable of integration was)



Integration of AN ALGEBRAIC FUNCTION

SUBSTITUTION METHOD

EXAMPLE 1: Integrate, $\int (6x-3)^4 dx$

Step 1

Let, $u = 6x - 3$

Step 2

Differentiate u, with respect to x

$$\frac{du}{dx} = 6 \mapsto dx = \frac{du}{6}$$

Step 3

Substitute u and dx into the original function

$$\int (6x-3)^4 dx = \int (u)^4 \cdot \frac{du}{6}$$

Step 4

Solve the integral

$$\int (6x-3)^4 dx = \frac{1}{6}\int (u)^4 \cdot du$$

$$= \frac{1}{6}\left[\frac{u^{4+1}}{4+1}\right] + c$$

$$= \frac{u^5}{6(5)} + c$$

Step 5

Substitute $u = 6x - 3$ for the final answer

$$\int (6x-3)^4 dx = \frac{u^5}{30} + c$$

$$= \frac{(6x-3)^5}{30} + c$$

EXAMPLE 2: Integrate, $\int 3k(6k^2-12)^5 dk$

Step 1

Let, $u = 6k^2 - 12$

Step 2

Differentiate u, with respect to x

$$\frac{du}{dk} = 12k \mapsto dk = \frac{du}{12k}$$

Step 3

Substitute u and dx into the original function

$$\int 3k(6k^2-12)^5 dk = \int \cancel{3}\cancel{k}(u)^5 \cdot \frac{du}{{}_4\cancel{12k}}$$

Step 4

Solve the integral

$$\int 3k(6k^2-12)^5 dk = \frac{1}{4}\int (u)^5 \cdot du$$

$$= \frac{1}{4}\left[\frac{u^{5+1}}{5+1}\right] + c$$

$$= \frac{u^6}{4(6)} + c$$

Step 5

Substitute $u = 6k^2 - 12$ for the final answer

$$\int 3k(6k^2-12)^5 dk = \frac{u^6}{24} + c$$

$$= \frac{(6k^2-12)^5}{24} + c$$

Integration of AN ALGEBRAIC FUNCTION

SUBSTITUTION METHOD

EXAMPLE 3: Integrate, $\int 3x(6x-12)^5 \, dx$

Step 1

Let, $u = 6x - 12$

Step 2

Differentiate u, with respect to x

$$\frac{du}{dx} = 6 \mapsto dx = \frac{du}{6}$$

Step 3

Substitute u and dx into the original function

$$\int 3x(6x-12)^5 \, dx = \int \cancel{3}x(u)^5 \cdot \frac{du}{\cancel{6}\,2}$$

$$= \frac{1}{2}\int x(u)^5 \cdot du$$

Step 4

Rewrite the equation in term of x

$u = 6x - 12$

$x = \frac{u+12}{6}$

Step 5

Substitute x into the function (step 3)

$$\int 3x(6x-12)^5 \, dx = \frac{1}{2}\int x(u)^5 \cdot du$$

$$= \frac{1}{2}\int \left(\frac{u+12}{6}\right)(u)^5 \cdot du$$

Step 6

Simplify the integral

expand

$$\int 3x(6x-12)^5 \, dk = \frac{1}{2}\int \left(\frac{u+12}{6}\right)(u)^5 \cdot du$$

$$= \frac{1}{2}\int \left(\frac{(u^6+12u^5)}{6}\right) \cdot du$$

Step 7

Solve the integral

$$\int 3x(6x-12)^5 \, dx = \frac{1}{12}\int (u^6 + 12u^5) \cdot du$$

$$= \frac{1}{12}\left[\frac{u^{6+1}}{6+1} + \frac{12u^{5+1}}{5+1}\right] + c$$

$$= \frac{1}{12}\left[\frac{u^7}{7} + \frac{12u^6}{6}\right] + c$$

Step 8

Substitute $u = 6x - 12$ for the final answer

$$\int 3x(6x-12)^5 \, dx = \frac{1}{12}\left[\frac{u^7}{7} + \frac{12u^6}{6}\right] + c$$

$$= \frac{1}{12}\left[\frac{(6x-12)^7}{7} + \frac{12(6x-12)^6}{6}\right] + c$$

Integration of AN ALGEBRAIC FUNCTION

SUBSTITUTION METHOD

Exercises | Integrate :

a] $\int (7x+8)^3\,dx$

b] $\int (3+8x)^7\,dx$

c] $\int (x+15)^{-3}\,dx$

d] $\int 4(3m+9)^3\,dm$

e] $\int \sqrt{(2s-7)^3}\,ds$

f] $\int \sqrt[5]{(3m-19)^3}\,dm$

g] $\int (8-x)^{2/3}\, dx$

h] $\int (2-x)^{1/5}\, dx$

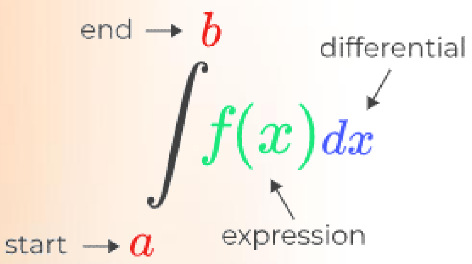
i] $\int \frac{2}{7}(x-5)^{-3}\, dx$

j] $\int \frac{5}{4(3m+2)^3} dm$

k] $\int \frac{8}{\sqrt{(2s-8)^3}} ds$

l] $\int \frac{9}{\sqrt[5]{(1-m)^3}} dm$

3.3 DEFINITE INTEGRAL



- the limit and summation,
- a and b (called limits, bounds or boundaries.

Definite Integral PROPERTIES

Properties of definite integrals	
1. Multiplication by a constant (k = constant)	$\int_a^b k{\cdot}f(x)\,dx = k\int_a^b f(x)\,dx$
2. Negation	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
3. Decomposition $a < c < b$	$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
4. Addition	$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
5. Zero integral	$\int_a^a f(x)\,dx = 0$

Integration of DEFINITE INTEGRAL

Example | Integrate:

a] $\int_{1}^{1} 3dx$

upper limit value

$= [3x]_1^2$ Lower limit value

$= [(3(2)) - (3(1))]$ [upper limit]-[lower limit]

$= [(6) - (3)]$

$= 3$

b] $\int_{-1}^{2}(4x - 5x^4)dx$

upper limit value

$= \left[\frac{4x^2}{2} - \frac{5x^5}{5}\right]_{-1}^{2}$

Lower limit value

$= [2x^2 - x^5]_{-1}^{2}$

$= [(2(2)^2 - (2)^5) - (2(-1)^2 - (-1)^5)]$

[upper limit]-[lower limit]

$= [(-24) - (3)]$

$= -27$

Formula method

c] $\int_{2}^{4}(4x - 6)^4 dx$

Power is +1

$= \left[\frac{(4x-6)^{4+1}}{4(4+1)}\right]_2^4$ upper limit value

Lower limit value

$\frac{dy}{dx}$ bracket New Power

$= \left[\frac{(4x-6)^5}{20}\right]_2^4$

$= \left[\left(\frac{(4(4)-6)^5}{20}\right) - \left(\frac{(4(2)-6)^5}{20}\right)\right]$

[upper limit]-[lower limit]

$= \left[\left(\frac{10^5}{20}\right) - \left(\frac{2^5}{20}\right)\right]$

$= \frac{24992}{5}$

Substitution method

d] $\int_{2}^{4}(4x - 6)^4 dx$

Let, $u = 4x - 6$

$\frac{du}{dx} = 4 \mapsto dx = \frac{du}{4}$

$= \frac{1}{4}\int_{2}^{10}(u)^4 \cdot du$

$= \frac{1}{4}\left[\frac{u^{4+1}}{4+1}\right]_2^{10}$

$= \frac{1}{4}\left[\frac{u^5}{5}\right]_2^{10}$

$= \frac{1}{20}[(10^5) - (2^5)]$

$= \frac{24992}{5}$

$x_1 = 4$

$= 4(4) - 6$
$= 10$

$x_2 = 2$

$= 4(2) - 6$
$= 2$

Integration of DEFINITE INTEGRAL

Exercises | Integrate:

a] $\int_{1}^{2} 3x\,dx$

b] $\int_{-1}^{2} 3x^7\,dx$

c] $\int_{3}^{7} 4x^{-2}\,dx$

d] $\int_{1}^{4} \frac{6x^2}{7}\,dx$

e] $\int_{2}^{3} x(1-x^2)dx$

f] $\int_{1}^{4} \frac{2x^2-7x+6}{5x^7}\,dx$

Formula method

Exercises | Integrate:

a] $\int_2^4 3(3-4m)^5 dm$

b] $\int_2^4 \sqrt[5]{(3s+9)^7} ds$

c] $\int_0^2 \frac{5}{\sqrt[3]{(1-m)}} dm$

d] $\int_1^2 \left(1-\frac{x}{2}\right)^5 dx$

Substitution method

Exercises | Integrate:

a] $\int_2^4 3(3-4m)^5 dm$

b] $\int_2^4 \sqrt[5]{(3s+9)^7}\, ds$

c] $\int_0^2 \frac{5}{\sqrt[3]{(1-m)}} dm$

d] $\int_1^2 \left(1-\frac{x}{2}\right)^5 dx$

e] $\int_1^2 x^2(3-5x^3)^2 dx$

f] $\int_1^2 x(x-3)^3 dx$

e] $\int_0^2 (2s+5)(3s-4)^2 ds$

d] $\int_1^2 \frac{s}{(1-3s^2)^2} ds$

3.4 TRIGONOMETRIC FUNCTION

$$\int \cos x \, dx = \sin x + c$$
$$\int \sin x \, dx = -\cos x + c$$
$$\int sec^2 x \, dx = \tan x + c$$

HOW TO INTEGRATE ?

STEPS:

- Integrate Trigo
- ( ) or bracket follow the question
- $\frac{dy}{dx}$( ) and divide
- + c (indefinite integral)

Remember!!!

1. Trigo function cannot have the power except $sec^2 x$
2. If Trigo function have the power, solve it by using trigonometry identities

$$\int sin(ax)dx = -\frac{cos(ax)}{a} + C$$

$$\int cos(ax)dx = \frac{sin(ax)}{a} + C$$

$$\int sec^2(ax)dx = \frac{tan(ax)}{a} + C$$

Bracket still same

$\frac{dy}{dx}$( )

Integration of TRIGONOMETRIC FUNCTION

Example | Integrate:

a] $\int 5\cos(2x)dx$

Integrate trigo

Bracket same

$$= \frac{5\sin(2x)}{(2)} + c$$

$\frac{dy}{dx}$ bracket

b] $\int 3\sin(2t+7)dt$

$$= \frac{3(-\sin(2t+7))}{(2)} + c$$

$$= \frac{-3\sin(2t+7)}{(2)} + c$$

c] $\int 8x\cos(8x^2-7)dx$

Substitution method

Let, $u = 8x^2 - 7$

$$\frac{du}{dx} = 16x \mapsto dx$$

$$= \frac{du}{16x}$$

$$= \frac{1}{2}\int \cos(u) \cdot du$$

$$= \frac{1}{2}\left(\frac{\sin(u)}{1}\right) + c$$

$\frac{dy}{dx}$ bracket

$$= \frac{\sin(8x^2-7)}{2} + c$$

Integration of TRIGONOMETRIC FUNCTION

Exercises | Integrate:

a] $\int \sin x \ dx$

b] $\int sec^2(4x)\, dx$

c] $\int 5cos(2x-7)\, dx$

d] $\int 18sin\left(\frac{1}{9}x\right)\, dx$

e] $\int \frac{3}{7}cos(7-x)\, dx$

f] $\int -4sin\left(\frac{2}{3}x+15\right)\, dx$

g] $\int \sin 3x - 7cos5x \ dx$

h] $\int sec^2(4x) - sin(x)\, dx$

i] $\int \sin(1-x) + 3cosx\, dx$

j] $\int 2x\sin(x^2)\,dx$

k] $\int -4x^3\sec^2(x^4)\,dx$

l] $\int 8x^2\cos(x^3+7)\,dx$

m] $\int \tan(2m)dm$

n] $\int (s-1)\sec^2(s^2-2s)ds$

o] $\int \tan^2\theta d\theta$

3.5 RECIPROCAL FUNCTION

$$\int \frac{1}{(x)} dx = ln(x) + c$$

$$\int \frac{1}{(ax+b)} dx = \frac{1}{a} ln(ax+b) + c$$

HOW TO INTEGRATE ?

$$\int \frac{1}{(x)} dx = ln(x) + c$$

Bracket still same

$$\int \frac{1}{(ax)} dx = \frac{1}{a} ln(ax) + c$$

$$\int \frac{1}{(ax+b)} dx = \frac{1}{a} ln(ax+b) + c$$

$\frac{dy}{dx}(\ \ )$

REMEMBER!!!

POWER cannot be '-1

Power of the **x or bracket** must be 1 (**denominator**)

Integration of RECIPROCAL FUNCTION

Example | Integrate:

a] $\int \frac{3}{2x} dx$

Power=1

Bracket same

$$= \frac{3 ln (x)}{2} + c$$

b] $\int \frac{4}{(1+3x)} dx$

Power=1

Bracket same

$$= \frac{4 \, ln \, (1+3x)}{(3)} + c$$

$\frac{dy}{dx}$ bracket

c] $\int \frac{2x}{x^2+4} dx$

Power of bracket =1

Substitution method

Let, $u = x^2 + 4$

$$\frac{du}{dx} = 2x \mapsto dx = \frac{du}{2x}$$

$$\int \frac{2x}{x^2+4} dx = \int \frac{2x}{u} \cdot \frac{du}{2x}$$

$$= \int \frac{1}{u} \cdot du$$

$$= \ln(u) + c$$

$$= \ln(x^2 + 4) + c$$

Integration of RECIPROCAL FUNCTION

Exercises | Integrate:

a] $\int \frac{2}{x}\, dx$

b] $\int \frac{3}{8x} dx$

c] $\int \frac{9}{8-7x} dx$

d] $\int \frac{x+2}{x^2-4} dx$

e] $\int \frac{4x^3+9x}{x^2}\, dx$

f] $\int \frac{4x^7-6x^3+9x}{2x^4} dx$

g] $\int \frac{m^2}{m^2+m^3} dm$

h] $\int \frac{1+t}{3+2t-t^2} dt$

i] $\int \frac{x^2-6}{2x^3-12x} dx$

j] $\int \frac{-s}{4s^2-7}\, ds$

k] $\int \frac{t^4}{6-t^5}\, dt$

l] $\int \frac{2m+3m^2}{m^2+m^3}\, dm$

m] $\int \frac{4m^2}{7-m^3}\, dm$

$n]$ $\int \frac{b-1}{b^2-1}\, db$

o] $\int \frac{s-3}{s^2-6s+5}\, ds$

3.6 EXPONENTIAL FUNCTION

$$\int e^{x}dx = e^{x} + c$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$$

HOW TO INTEGRATE ?

$$\int e^{x}dx = e^{x} + c$$

Exp & power still same

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$$

$\frac{dy}{dx}$ *power of exp*

INTEGRATE EXPONENTIAL FUNCTION

- Exponent and the power cannot change (copy from question)
- $\frac{dy}{dx}$ power of exponent and put at denominator
- + c (indefinite integral)

REMEMBER!!!

Exponent function UNCHANGED

Integration of EXPONENTIAL FUNCTION

Example | Integrate:

a] $\int e^{-3x} dx$

Exponent & power unchanged

$$= \frac{e^{-3x}}{-3} + c$$

$\frac{dy}{dx}$ *power of exp*

b] $\int 4e^{5x+9} dx$

Exponent & power unchanged

$$= \frac{4e^{5x+9}}{5} + c$$

$\frac{dy}{dx}$ *power of exp*

c] $\int \frac{4e^{2x}}{(2+e^{2x})^8} dx$

Substitution method

Let, $u = 2 + e^{2x}$

$$\frac{du}{dx} = 2e^{2x} \mapsto dx = \frac{du}{2e^{2x}}$$

$$\int \frac{4e^{2x}}{(2+e^{2x})^8} dx = \int \frac{4e^{2x}}{u^8} \cdot \frac{du}{2e^{2x}}$$

$$= \int 2u^{-8} \cdot du$$

$$= 2\,\frac{u^{-7}}{-7} + c$$

$$= 2\,\frac{1}{-7u^7} + c$$

$$= \frac{2}{-7(2+e^{2x})^7} + C$$

Integration of EXPONENTIAL FUNCTION

Exercises | Integrate:

a] $\int e^{-t}\, dt$

b] $\int 3e^{-x^2} dx$

c] $\int -3e^{-\frac{x}{3}} dx$

d] $\int e^{-2x-7} dx$

e] $\int e^{-(3x-6x^2)}\, dx$

f] $\int 2e^{\sin 3x} dx$

g] $\int e^{-4m+7} - 9e^{2m}\, \mathrm{dm}$

h] $\int e^{-t+4} - 3e^{-t} dt$

i] $\int 5x^2 e^{x^3} dx$

j] $\int \frac{x^4}{e^{x^5}} dx$

k] $\int \frac{2e^{\sqrt{t}}}{\sqrt{t}} dt$

l] $\int e^m (1 + e^m)^4 dm$

m] $\int \frac{4e^{-3m}}{(2+e^{-3m})^8} dm$

n] $\int \frac{\left(8+e^{-b}\right)^7}{e^b} db$

o] $\int \frac{e^s}{\sqrt{e^s+2}} ds$

3.7 BY PART

$$\int udv = uv - \int vdu$$

PRIORITY OF U

L - logarithmic functions

A - algebraic (polynomials)

T - trigonometric functions

E - exponential functions

HOW TO INTEGRATE ?

STEPS:

- Find u & $\int dv$

  # Check the priority for u

- Differentiate u then make

  du as subject matter

- Integrate $\int dv \rightarrow$ find v
- Insert u , v into the formula

$$uv - \int vdu$$

- Integrate $\int vdu$
- Solve & simplify

$$\int xsinx\ dx$$

$u = x$ $\qquad$ $\int dv = \int sinxdx$

$\frac{du}{dx} = 1$ $\qquad$ v= $-cosx$

$du = dx$

Formula

$$\int udv = uv - \int v\ du$$

$$\int xsinxdx = x(-cosx) - \int (-cosx)dx$$

$$\int xsinxdx = -xcos\ x + \int (cosx)dx$$

$$\int xsinxdx = -xcos\ x + \sin x + c$$

BY PART

TABULAR METHOD

Sign	f(x)-Differentiate (LATE-PRIOTY)	f(y) Integration
+	f(x)	f(y)
-	First derivative of f(x)	First Integrate of f(y)
+	Second derivative of f(x)	Second Integrate of f(y)
-	Third derivative of f(x)- until zero	third Integrate of f(y)

$\int dx$

STEPS:

- Make a table with 3 column $(+/-\ ,\ \frac{dy}{dx}\ and\ \int dv)$
- Multiply sign and F(x) with the first integration of F(y)
- Multiply the first derivative of F(x) with the second integration of F(y)

  ....and so on.
- Multiply the last derivative f(x) and f(y) and integrate $\int dv$
- Add all of them

Remember!!!

Logarithm -differentiate one time only

Algebraic-differentiate repeatedly until you obtain 0

Combination trigonometric & exponential -differentiate until second derivative

Integration of BY PART

Example | Solve the problem:

$$a]\int x^2 \ln(x)\, dx$$

<table>
<tr><th>Formula by part</th><th>Tabular method</th></tr>
<tr><td>

$\int x^2 ln(x) dx$

$u = ln(x)$ $\int dv = \int x^2 dx$

$\frac{du}{dx} = \frac{1}{x}$ $v = \frac{x^3}{3}$

$du = \frac{dx}{x}$

Formula

$\int udv = uv - \int v\, du$

$= ln(x)\left(\frac{x^3}{3}\right) - \int \left(\frac{x^3}{3}\right)\frac{dx}{x}$

$= \left(\frac{x^3}{3}\right) ln(x) - \int \left(\frac{x^2}{3}\right) dx$

$= \left(\frac{x^3}{3}\right) ln(x) - \frac{x^3}{9} + c$

</td><td>

sign	Differentiation	Integration
+	$ln(x)$	x^2
-	$\frac{1}{x}$	$\frac{x^3}{3}$

$\int dx$

$\int x^2 ln(x) dx = \frac{x^3}{3} ln(x) + \int \left(-\frac{1}{x}\right)\left(\frac{x^3}{3}\right) dx$

$= \left(\frac{x^3}{3}\right) ln(x) - \int \left(\frac{x^2}{3}\right) dx$

$= \left(\frac{x^3}{3}\right) ln(x) - \frac{x^3}{9} + c$

</td></tr>
</table>

Integration of BY PART

Example | Solve the problem:

b] $\int 3x^2 e^{-x} dx$

Formula by part

$$\int 3x^2 e^{-x} dx$$

$u = 3x^2$

$\frac{du}{dx} = 6x$

$du = 6x dx$

$\int dv = \int e^{-x} dx$

v= $-e^{-x}$

Formula $\int u dv = uv - \int v\, du$

$=-3x^2 e^{-x} - \int -e^{-x} (6x dx)$

$=-3x^2 e^{-x} + \int 6x e^{-x} dx$

solve using by part again

$u = 6x$

$\frac{du}{dx} = 6$

$du = 6dx$

$\int dv = \int e^{-x} dx$

v= $-e^{-x}$

Formula

$\int u dv = uv - \int v\, du$

$=-6x e^{-x} - \int -e^{-x} (6dx)$

$=-6x e^{-x} + \int 6 e^{-x} dx$

$=-6x e^{-x} - 6e^{-x} + \mathrm{c}$

$$\int 3x^2 e^{-x} dx$$

$=-3x^2 e^{-x} - 6x e^{-x} - 6e^{-x} + \mathrm{c}$

Tabular method

sign	$\frac{dy}{dx}$	$\int dx$
+	$3x^2$	e^{-x}
-	$6x$	$-e^{-x}$
+	6	e^{-x}
-	0	$-e^{-x}$

$\int dx = 0$

$$\int 3x^2 e^{-x} dx$$

$=-3x^2 e^{-x} - 6x e^{-x} - 6e^{-x} + c$

Integration of BY PART

Example | Solve the problem:

c] $\int 3e^x sinx dx$

Formula by part	Tabular method
$\int 3e^x sinxdx$ $u = 3sinx$ $\quad \int dv = \int e^x dx$ $\frac{du}{dx} = 3cosx$ $\quad$ v= e^x $du = 3cosxdx$ Formula $\int udv = uv - \int v\, du$ $\int 3e^x sinxdx = 3sinxe^x - \int 3e^x cosxdx$ solve using by part again $u = 3cosx$ $\frac{du}{dx} = -3\sin x$ $\quad \int dv = \int e^x dx$ $du = -3sinx\, dx$ $\quad$ v= e^x $\int 3e^x sinxdx = 3e^x sinx - \int 3e^x cosxdx$ $\int 3e^x sinxdx = 3e^x sinx - \left(uv - \int v\, du\right)$ $= 3e^x sinx - (3e^x cosx - \int e^x(-3sinx)dx)$ $= 3e^x sinx - (3e^x cosx + \int 3e^x sin\, xdx)$ $= 3e^x sinx - 3e^x cosx - \int 3e^x sin\, xdx$ Same with question $\int 3e^x sinxdx + \int 3e^x sin\, xdx = 3sinxe^x - 3e^x cosx$ $\int 6e^x sinxdx = 3e^x(sinx - cosx)$ $\int e^x sinxdx = \frac{e^x}{2}(sinx - cosx)$+c	<table><tr><th>sign</th><th>$\frac{dy}{dx}$</th><th>$\int dx$</th></tr><tr><td>+</td><td>➡ 3sin x</td><td>e^x</td></tr><tr><td>-</td><td>➡ $3cosx$</td><td>e^x</td></tr><tr><td>+</td><td>➡ $-sinx$</td><td>e^x</td></tr></table> $\xrightarrow{\int dx}$ $\int 3e^x sinxdx = 3sinxe^x - 3e^x cosx$ $- \int 3e^x sin\, xdx$ Same with question $\int 3e^x sinxdx + \int 3e^x sin\, xdx$ $= 3sinxe^x - 3e^x cosx$ $\int 6e^x sinxdx = 3e^x(sinx - cosx)$ $\int e^x sinxdx = \frac{e^x}{2}(sinx - cosx)$+c

Integration BY PART

Exercises | Solve the problem:

$a] \int 3t^2 ln(4t)dt$

(Formula method)
(Tabular method)

b] $\int x\ln(4x)dx$

(Formula method)
(Tabular method)

c] $\int 3x^2 e^{2x+1} dx$

(Formula method)

(Tabular method)

d] $\int 8x^3e^{-2x}dx$

(Formula method)

(Tabular method)

e] $\int 8x^3 e^{3x} dx$

(Formula method)

(Tabular method)

f] $\int x^3\cos(4x+2)dx$

(Formula method)

(Tabular method)

g] $\int 2e^{x}\cos(x)dx$

(Formula method)

(tabular method)

h] $\int 2e^{2x}\sin(x)dx$

(Formula method)

(Tabular method)

i] $\int 4e^{3x}cos(2x)dx$

(Formula method)

(Tabular method)

j] $\int e^{-x} cos(2x+3)dx$

(Formula method)

(Tabular method)

3.8 PARTIAL FRACTION

Type	*Factor example*	*Decomposition*
Linear factor	$(x-4)$	$\frac{A}{x-4}$
Repeated linear factor	$(x-4)^2$	$\frac{A}{(x-4)}+\frac{B}{(x-4)^2}$
Quadratic irreducible factor	(x^2+4)	$\frac{Ax+B}{(x^2+4)}$

HOW TO INTEGRATE ?

STEPS:

- Factor the bottom of the fraction-setup equation

$$\frac{5x-4}{x^2-x-2} = \frac{5x-4}{(x-2)(x+1)}$$

- Decompose into partial fraction factors-linear factor/quadratic factor/ repeated factor

$$\frac{5x-4}{(x-2)(x+1)} = \frac{A_1}{x-2} + \frac{A_2}{x+1}$$

- Multiply through by the bottom so we no longer have fractions

$$5x-4 = A_1(x+1) + A_2(x-2)$$

- find the constants A and B (calculator also can)

Root for (x+1) is x = -1

$$5(-1)-4 = A_1(-1+1) + A_2(-1-2)$$
$$-9 = 0 + A_2(-3)$$
$$A_2 = 3$$

Root for (x-2) is x = 2

$$5(2)-4 = A_1(2+1) + A_2(2-2)$$
$$6 = A_1(3) + 0$$
$$A_1 = 2$$

- Replace the values of A and B into the equationType equation here.

$$\frac{5x-4}{x^2-x-2} = \frac{2}{x-2} + \frac{3}{x+1}$$

- Integrate the equation

$$\int \frac{5x-4}{x^2-x-2}\,\mathrm{dx} = \int \frac{2}{x-2}\,dx + \int \frac{3}{x+1}\,dx$$

Integration by PARTIAL FRACTION

Example | Solve the problem:

$$\int \frac{3x+4}{x^2-4}\,dx$$

Step 1

Factor and decompose the bott om of fraction,

$$\int \frac{3x+4}{x^2-4}\,dx = \int \frac{3x+4}{(x-2)(x+2)}\,dx$$

Linear factor

Step 2

Decompose into partial fraction

$$\frac{3x+4}{x^2-4} = \frac{3x+4}{(x-2)(x+2)}$$

$$= \frac{A}{(x-2)} + \frac{B}{(x+2)}$$

Step 3

Multiply through by the bottom

$$3x+4 = A(x+2) + B(x-2)$$

$$3x+4 = Ax + 2A + Bx - 2B$$

Step 4

Find the constant A and B (using calculator)

$$3x+4 = Ax + 2A + Bx - 2B$$

Coeffecient

x → $A + B = 3$

Null → $2A - 2B = 4$

$$A = \frac{5}{2} \qquad B = \frac{1}{2}$$

Step 5

Replace the values of A and B

$$\frac{3x+4}{x^2-4} = \frac{5}{2(x-2)} + \frac{1}{2(x+2)}$$

Step 6

Integrate the equation

$$\int \frac{3x+4}{x^2-4}\,dx = \int \frac{5}{2(x-2)}\,dx + \int \frac{1}{2(x+2)}\,dx$$

$$\int \frac{3x+4}{x^2-4}\,dx = \frac{5}{2}\ln(x-2) + \frac{1}{2}\ln(x+2) + c$$

Exercises | Solve the problem:

a] $\int \frac{2x}{x^2+x-2}\, dx$

b] $\int \frac{3x-5}{x^2-x-2}\, dx$

c] $\int \frac{b}{2b^2-b-3} \, db$

d] $\int \frac{14m+7}{(m+1)^2(2m-5)}\ dm$

e] $\int \frac{11m+17}{2m^2+7m-4}\, dm$

f] $\int \frac{3t-5}{t^2-t-2}\, dt$

$g] \int \frac{x^3+2}{(x-2)(x+3)}\,dx$

h] $\int \frac{11x + 17}{2x^2 + 7x - 4} dx$

i] $\int \frac{-t^2 + 3t + 1}{(t+1)(t^2+2)} dt$

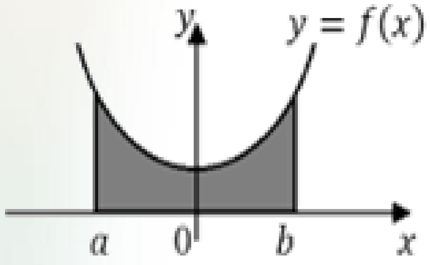
j] $\int \frac{t^3+1}{t^2-t}\,dt$

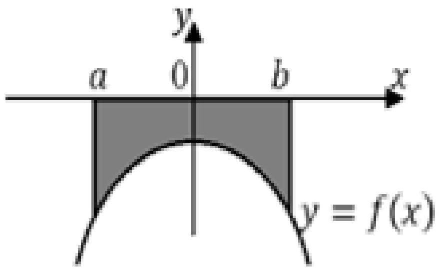
k] $\int \frac{x^3+1}{(x^2-x)} dx$

3.9 APPLICATION OF INTEGRATION

x-axis curve

AREA	VOLUME
$A_x = \int_a^b y\ dx$	$V_x = \pi \int_a^b x^2\ dx$



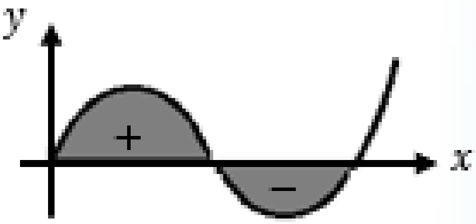


REMEMBER!!!

When used $\int y\ dx$

Negative value below x-axis

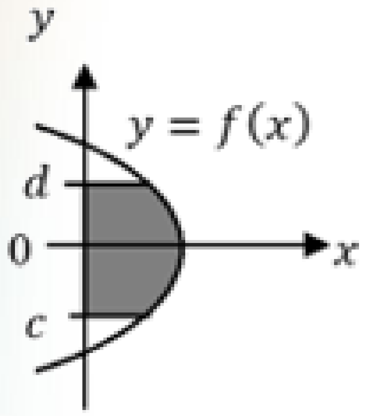
Positive value on the x-axis

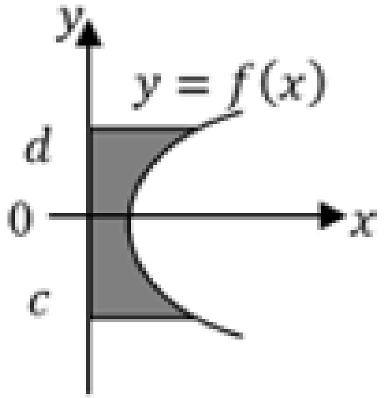


3.9 APPLICATION OF INTEGRATION

y-axis curve

AREA	VOLUME
$A_y = \int_c^d x \, dy$	$V_y = \pi \int_c^d x^2 \, dy$



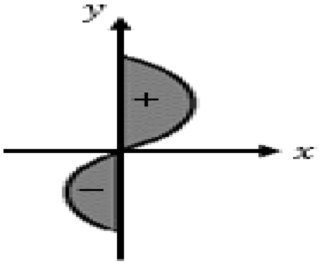


REMEMBER!!!

When used $\int x \, dy$

Negative value at the left y-axis

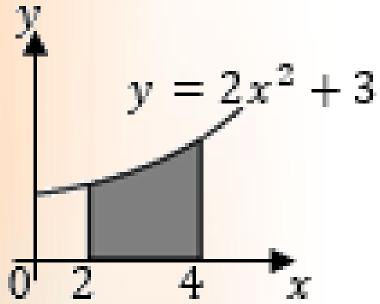
Positive value at the right y-axis



Integration by APPLICATION OF INTEGRATION

Example | Find the area and volume bounded for x-axis:

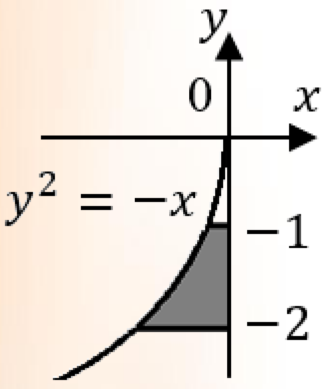
a]



AREA	VOLUME
$A_x = \int_a^b y\ dx$ $= \int_2^4 (2x^2+3)dx$ $= \left[\frac{2x^3}{3} + 3x\right]_2^4$ $= \left(\frac{2(4)^3}{3} + 3(4)\right) - \left(\frac{2(2)^3}{3} + 3(2)\right)$ $= \frac{130}{3} unit^2$	$V_x = \pi \int_a^b x^2\ dx$ $= \pi \int_2^4 (2x^2+3)^2 dx$ $= \pi \int_2^4 (4x^4 + 12x^2 + 9)dx$ $= \pi \left[\frac{4x^5}{5} + \frac{12x^3}{3} + 9x\right]_2^4$ $= \pi \left[\left(\frac{4(4)^5}{5} + \frac{12(4)^3}{3} + 9(4)\right) - \left(\frac{4(2)^5}{5} + \frac{12(2)^3}{3} + 9(2)\right)\right]$ $= \frac{5178}{5} \pi\ unit^3$

Example | Find the area and volume bounded for y-axis:

b]



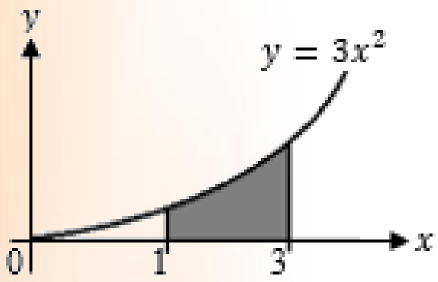
Area & volume has no negative value. A negative sign indicates that the shaded region is below the x-axis

AREA	VOLUME

Integration by APPLICATION OF INTEGRATION

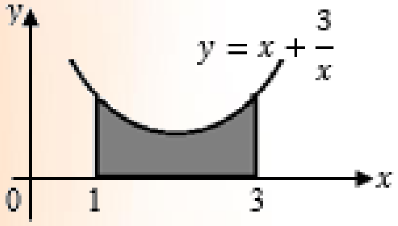
Example | Find the area and volume bounded :

a]



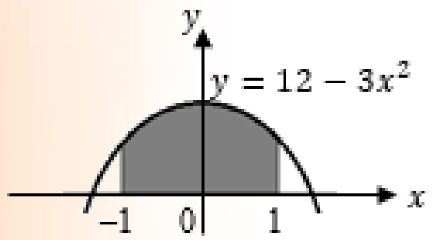
AREA	VOLUME

b]



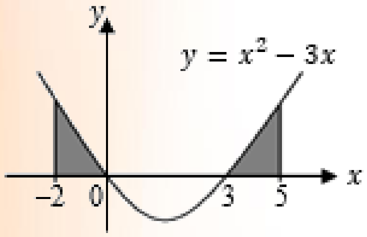
AREA	VOLUME

c]



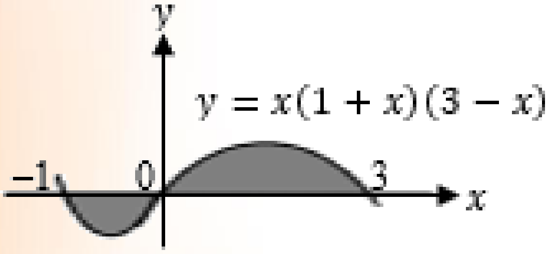
AREA	VOLUME

d]



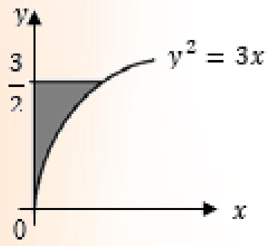
AREA	VOLUME

e]



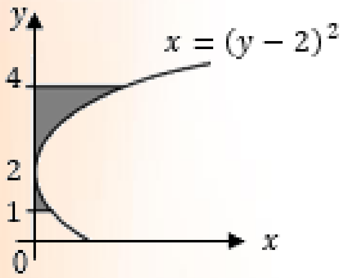
AREA	VOLUME

f]



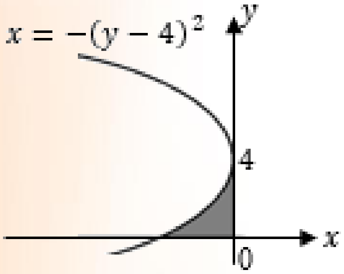
AREA	VOLUME

g]



AREA	VOLUME

h]

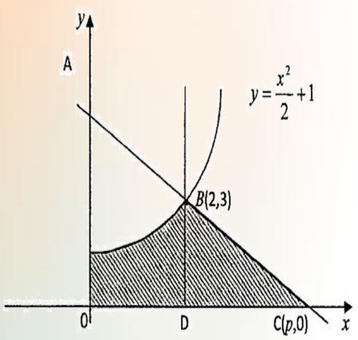
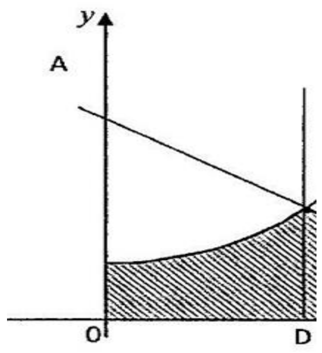
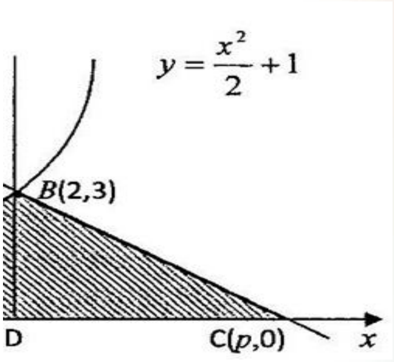
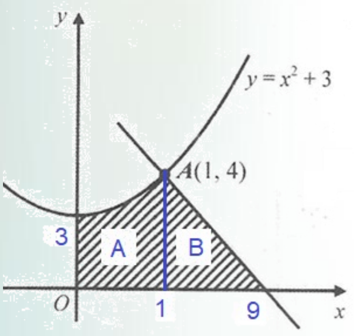
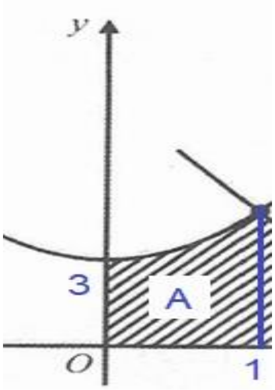
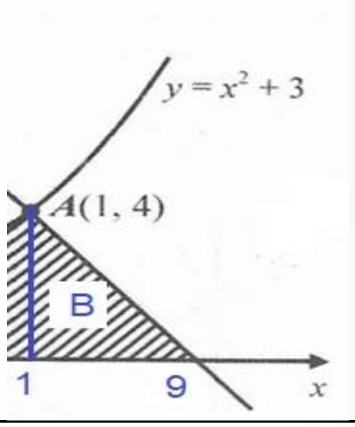
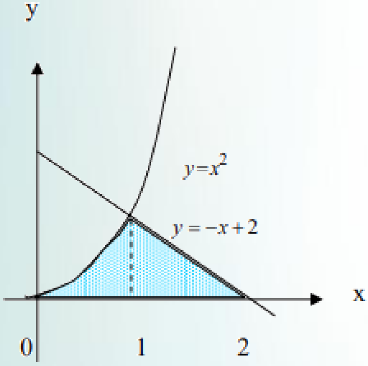
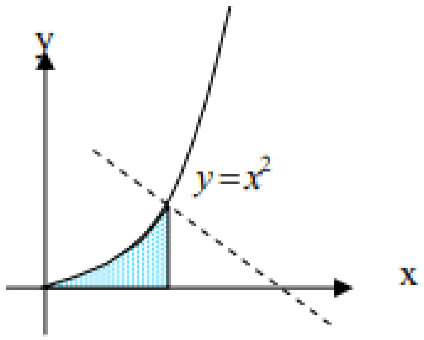
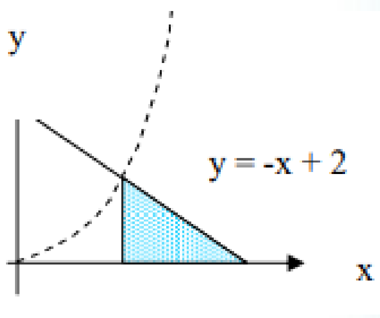


AREA	VOLUME

Integration by APPLICATION OF INTEGRATION

Area and volume under the curve (ADDITION)

ATTENTION: A = B + C

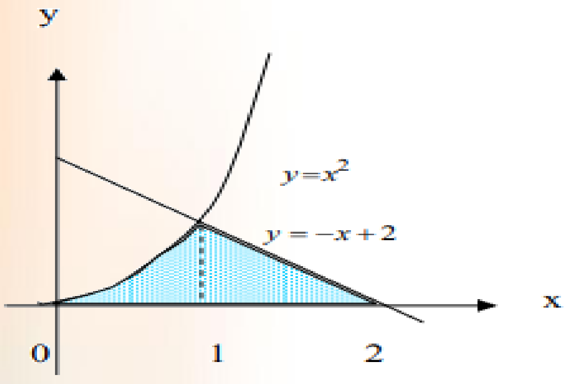
A	B	C
		
		
		

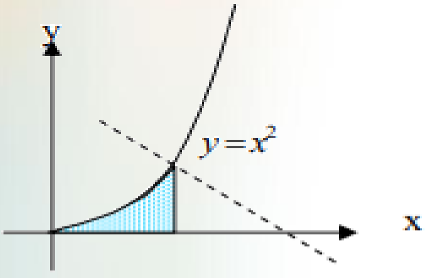
REMEMBER!!!
The Limit Are Not Same

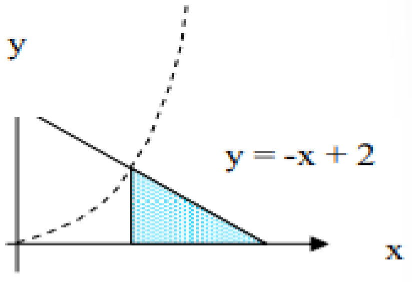
Integration by APPLICATION OF INTEGRATION

Example | **Find the area bounded for x-axis:**

a]

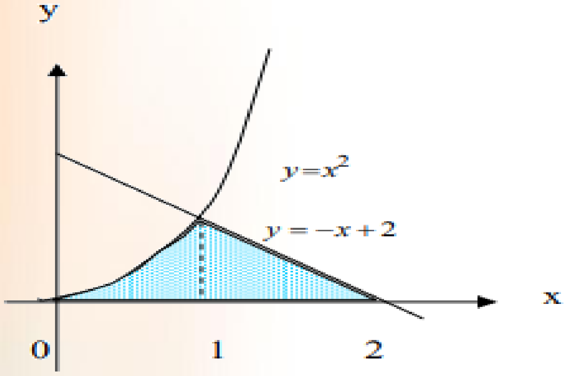


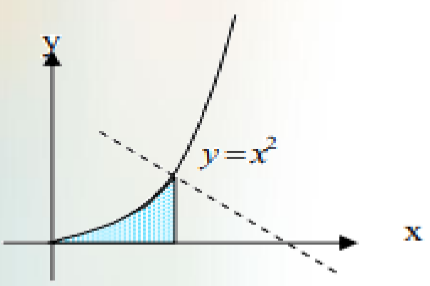


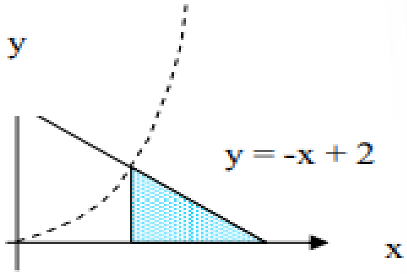


Example | Find the volume bounded for x-axis:

b]



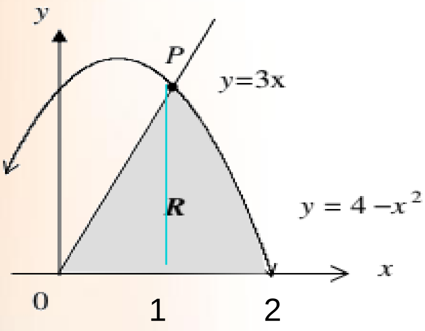




Integration by APPLICATION OF INTEGRATION

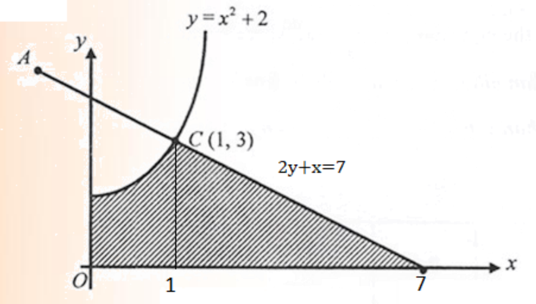
Example | Find the area and volume bounded :

a]



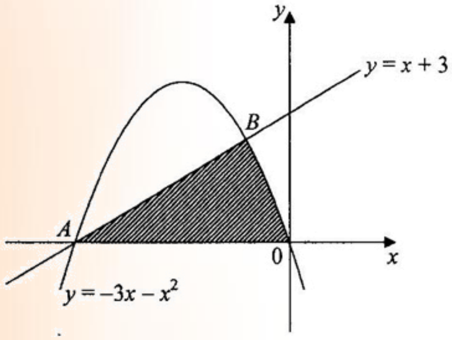
Area:
Volume:

b]



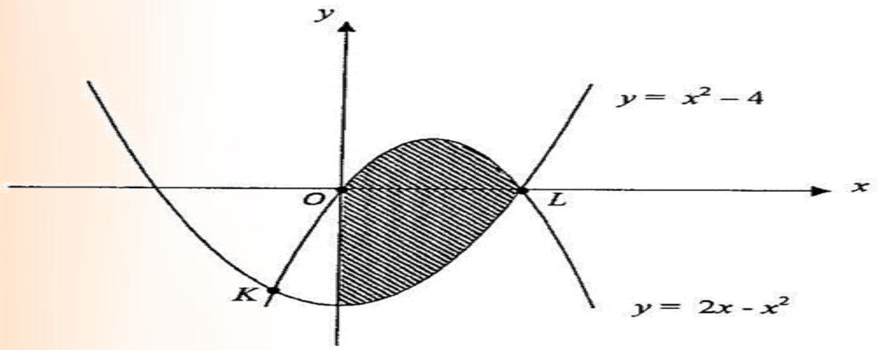
Area:
Volume:

c]



Area:
Volume:

d]

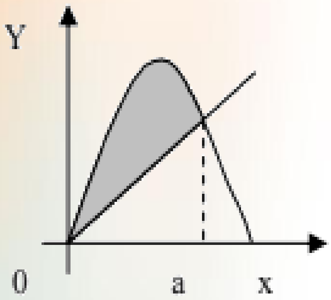
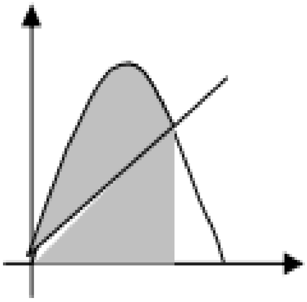
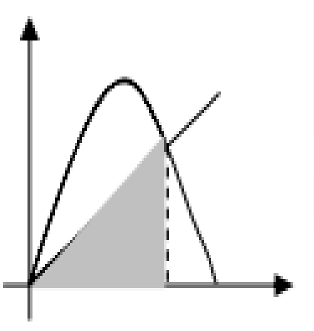
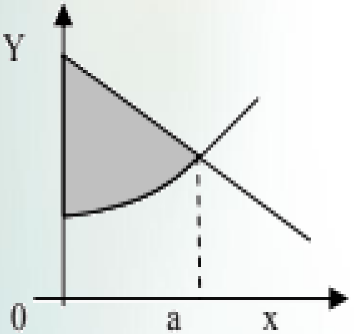
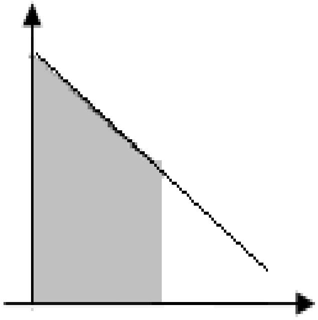
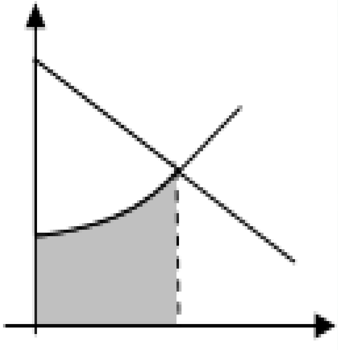
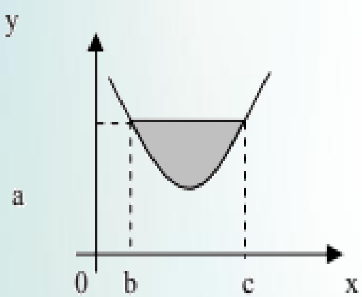
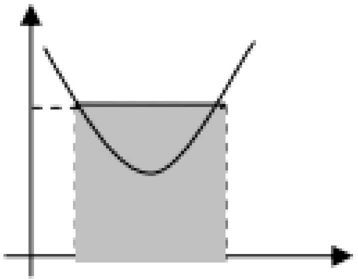
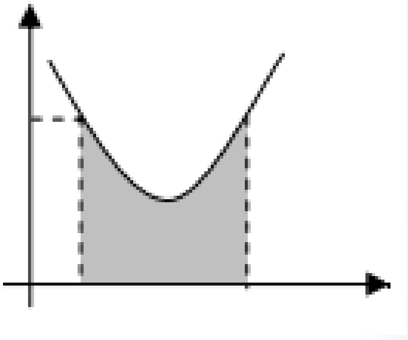


Area:
Volume:

Integration by APPLICATION OF INTEGRATION

Area and volume under the curve (SUBTRACTION)

ATTENTION: A = B - C

A	B	C
		
		
		

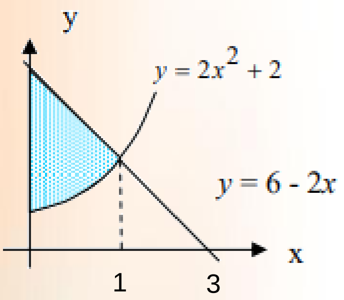
REMEMBER!!!

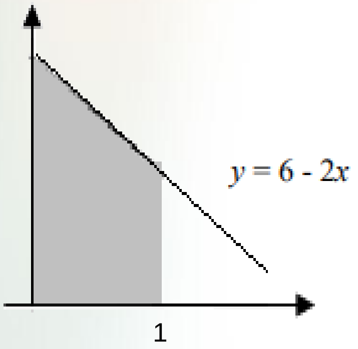
The Limit Are Same

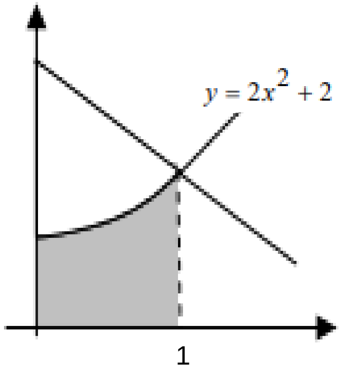
Integration by APPLICATION OF INTEGRATION

Example | Find the area bounded for x-axis:

a]

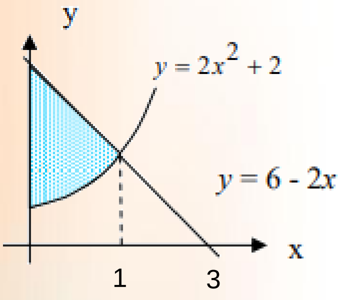


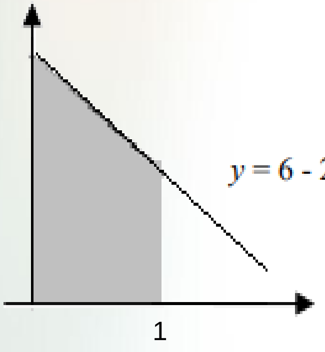


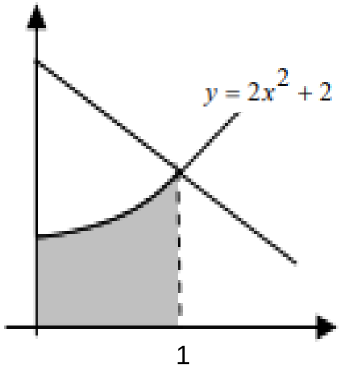


Integration by APPLICATION OF INTEGRATION

Example | Find the volume bounded for x-axis:

b] 

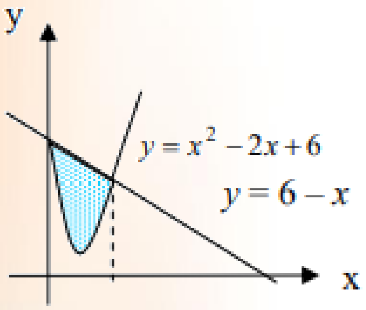




Integration by APPLICATION OF INTEGRATION

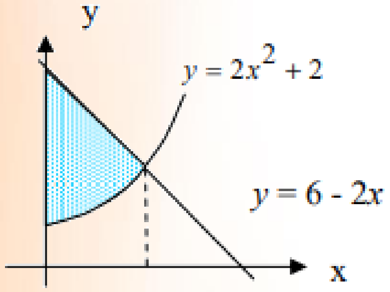
Exercises | Find the area and volume bounded :

a]



Area:
Volume:

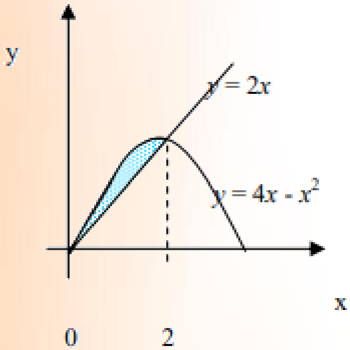
b]



Area:

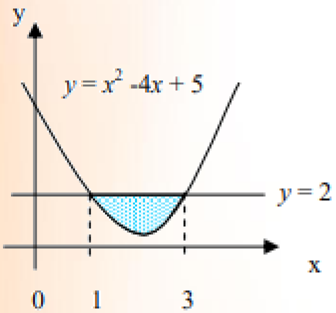
Volume:

c]



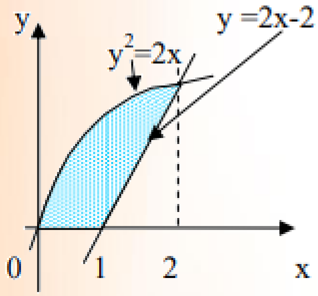
Area:
Volume:

d]



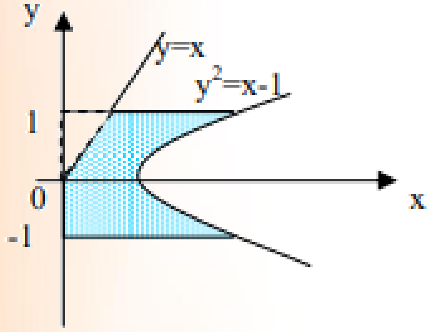
Area:
Volume:

e]



Area:
Volume:

f]



Area:

Volume:

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