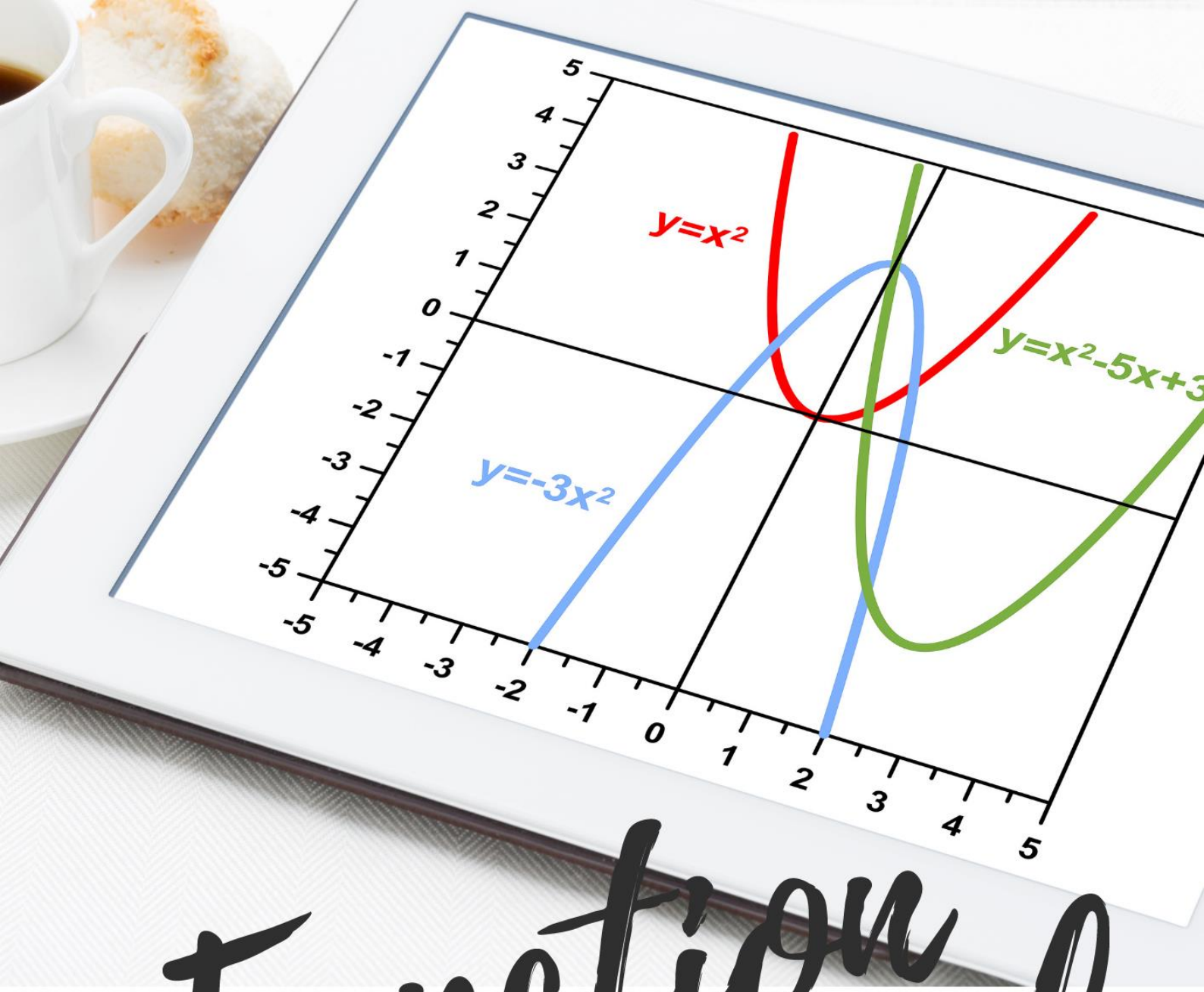

Function and graph

A STUDENT'S HANDBOOK

NOREHAN-HIDAYAH-AZNA

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Writter:

Norehan binti Abu Hassan

Nur Hidayah binti Khalid

Nurul Azna Idayu binti Ab Aziz

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Kompleks Pendidikan Bandar Enstek,

71760, Bandar Enstek, Negeri Sembilan

PREFACE

FUNCTION AND GRAPH – A Student's handbook is written as a reference for student enrolled in course DBM10133- Mathematics for Technology at Polytechnics Malaysia. The book contains three subtopics: Linear Function, Quadratic Function and Simultaneous Equation. The notes provided in this book have been written based on syllabus of the course and supported with diagrams for better understanding the topics. On top of that, the practices and assessment provided in this book are tailored to suits the needs of students in mastering the topics and the course.

Table of Contents

4.1: Linear Function

Learning Outcome

By the end of the lesson student should be able to:

1. Define graphs.
2. Plot graphs by using table of values.
3. Draw a linear graph.
4. Find midpoint, distance and gradient between two points.

INTRODUCTION

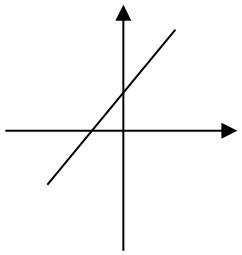
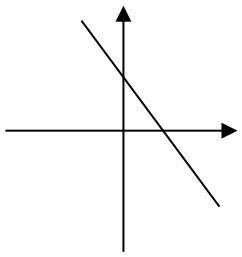
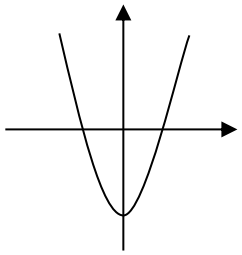
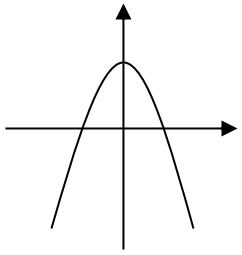
A graph of a function is a representation of the function by means of a line in a Cartesian plane. Graph of *y* versus *x*, means that the *y* and *x* are two variables. The vertical line is *y* axis and the horizontal line is *x* axis.

Function is a relationship involving two variables where one of the variables is written in terms of order variables.

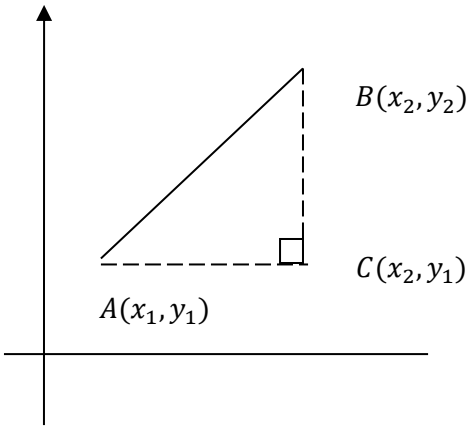
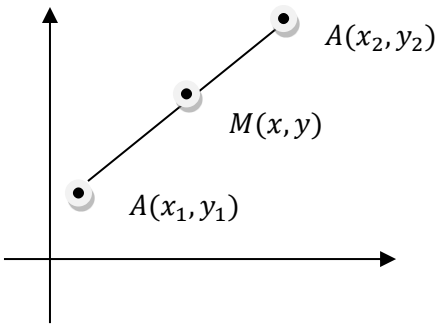
A table of values for a given function with two variables is a table consisting of the variables.

A table of values for a given function can be constructed substituting each value of *x* into the equation of the function to obtain the corresponding value of *y*.

TYPES OF GRAPHS

Name	Description	Figure	
		a > 0	a < 0
Linear graphs	The power for the independent variables is 1. The general form of linear graph is $\boldsymbol{y = mx + c}$		
Quadratic graphs	The highest power for the independent variables *x* is 2. The general form of a quadratic equation is $\boldsymbol{y = ax^2 + bx + c}$ c where *a*, *b* and *c* are real numbers and *a* is not equal to zero		

GRAPH OF LINEAR FUNCTIONS

Name	Formula	Description
Equation	$y = mx + c$	***y*** and ***x*** are two variables, ***c*** is a coordinate of *y*-intercept and ***m*** is a gradient of the straight line.
Gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$	
Distance	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
Midpoint	$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	

EXAMPLE 1

Given point A $(-2,0)$ and point B $(0,4)$. Find

i. the gradient of point A and B
ii. the distance of point A and B
iii. the midpoint of point A and B

Solution

gradient	distance	midpoint
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{4-0}{0-(-2)}$ $m = \frac{4}{2}$ $= 2$	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(0-(-2)^2 + (4-0)^2}$ $AB = \sqrt{2^2 + 4^2}$ $AB = \sqrt{20} = 4.47$	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $M = \left(\frac{-2+0}{2}, \frac{0+4}{2}\right)$ $M = \left(\frac{-2}{2}, \frac{4}{2}\right) = (-1,2)$

EXAMPLE 2

Given point J $(4,2)$ and point K $(3,6)$. Find

i. the gradient of point J and K
ii. the distance of point J and K
iii. the midpoint of point J and K

Solution

gradient	distance	midpoint
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6-2}{3-4}$ $m = \frac{4}{-1}$ $= -4$	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(3-4)^2 + (6-2)^2}$ $AB = \sqrt{(-1)^2 + 4^2}$ $AB = \sqrt{17} = 4.12$	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $M = \left(\frac{4+3}{2}, \frac{2+6}{2}\right)$ $M = \left(\frac{7}{2}, \frac{8}{2}\right) = (\frac{7}{2}, 4)$

DRAWING A LINEAR GRAPH

When drawing graph of functions using a given scale, the following steps are taken:

1st : Construct a table of values for the function by choosing suitable numbers i.e. integers wherever possible so that it is easier to plot the graph.

2nd: Draw the axes i.e. x-axis and y-axis and mark the scales accordingly i.e. 1cm : 5 units for y-axis. In cases where scales are not given, then we must choose suitable scales for both axes.

3th: Write down the origin point as 0 and the arrow for both axes.

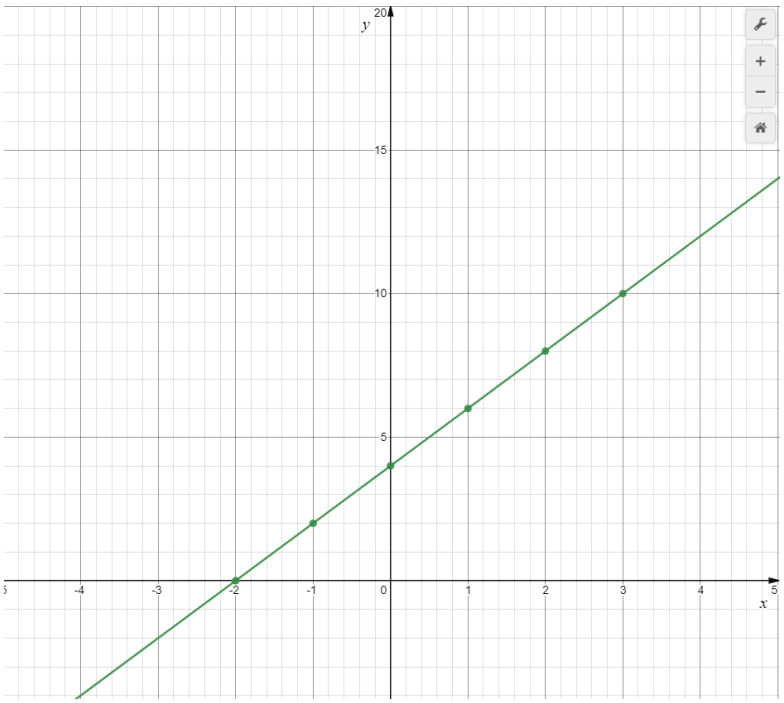
4th: Join the points and label the graph.

EXAMPLE 1

Draw a graph for $y = 2x + 4$

1st : Construct a table of values for the function by choosing suitable numbers i.e. integers wherever possible so that it is easier to plot the graph.

x	-2	-1	0	1	2	3
y	0	2	4	6	8	10

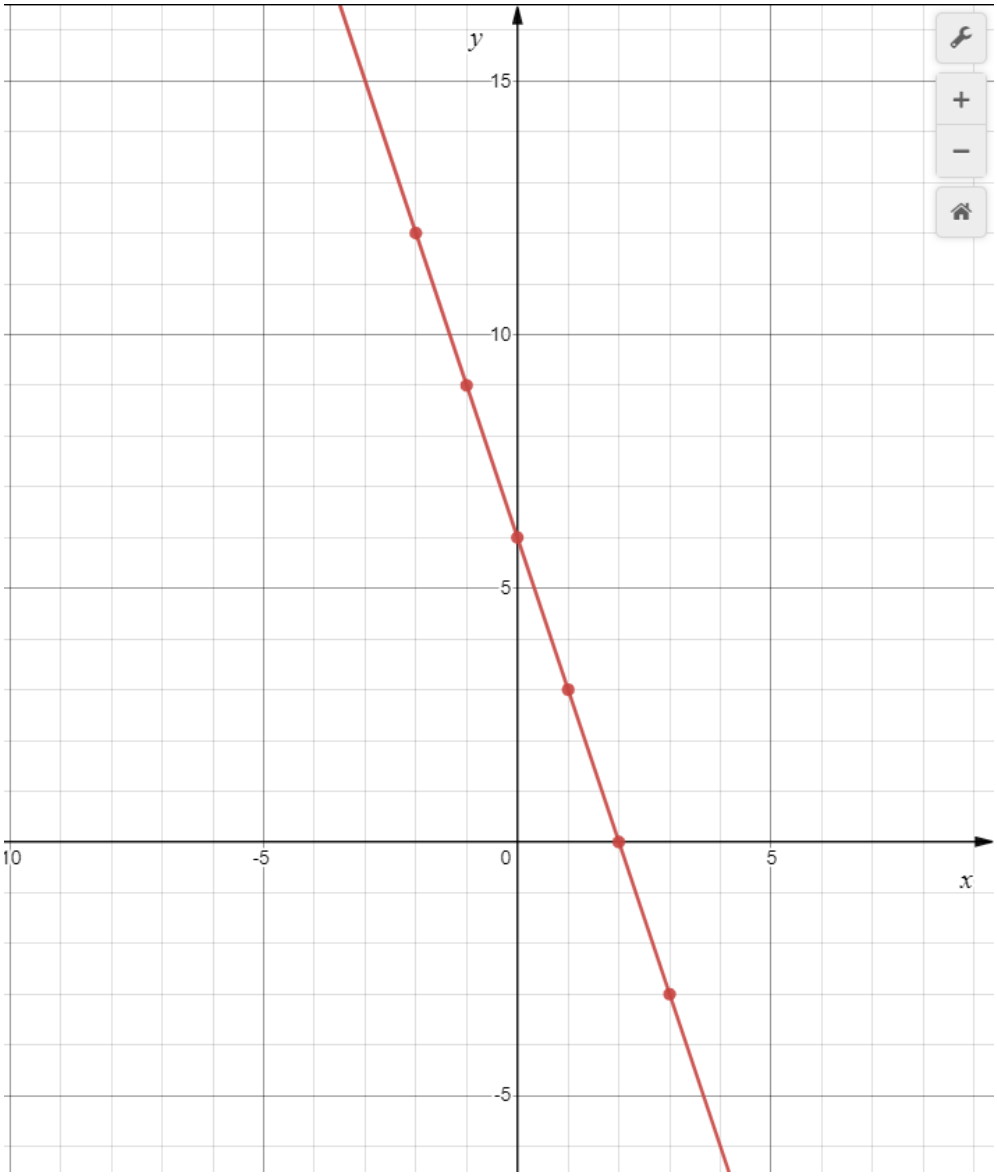


EXAMPLE 2

Draw a graph for $y = -3x + 6$

1st : Construct a table of values for the function by choosing suitable numbers i.e. integers wherever possible so that it is easier to plot the graph.

x	-2	-1	0	1	2	3
y	12	9	6	3	0	-3



EXERCISE 4.1

1. Determine the gradient and y-intercept in the following linear function:

   i. $y = 4x - 3$
   ii. $4x = -8x + 16$
   iii. $7x + y = 5$
   iv. $3x + 5y = -5$
   v. $2x + y = 4$

2. Find the gradient, distance, and midpoint for each of the set data point below:

   i. A$(-4,7)$ and B$(-6,-4)$
   ii. M$(3,0)$ and N$(-11,-15)$
   iii. J$(3,-20)$ and K$(5,8)$
   iv. X$(1,-19)$ and Y$(-2,-7)$
   v. E$(12,2)$ and Y$(-7,5)$

3. By referring to the following tables of values, draw a graph *y* versus *x* for $y = 3x - 6$

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
y	-6	-4	-3	-2	0	2	3	4	6	8

4. Complete the following table of values for the function $y = -2x + 8$, then sketch the graph.

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
y										

3. Draw the graph with the equations given below:

   i. $y = x + 3$
   ii. $y = x - 4$
   iii. $y = 3x + 1$
   iv. $y = -x + 4$
   v. $y = 8 - 2x$

4.2: Quadratic Function

Learning Outcome

By the end of the lesson student should be able to:

1. Plot the graph of a quadratic function
2. Drawing a graph of quadratic function

INTRODUCTION

A quadratic function is of the form $y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$.

For a quadratic function in the form of $y = ax^2 + bx + c$, the value of c is always the y-intercept of the graph.

DRAWING A QUADRATIC GRAPH

Steps to draw a graph:

1st Shape of the graph

-by looking at the value of a

- if the value of a>0, the graph will be minimum graph.
- If the value of a<0, the graph will be maximum graph.

2nd: Find y-intercept when $x = 0$

- Will get the value of c

3rd : Find x intercept when $y = 0$

4th: Find the vertex ; Vertex (h, k)

- $h = \frac{-b}{2a}$
- $k = f(h)$

5th : Build tables of values

6th: Sketch the graph.

EXAMPLE 1

Sketch a graph of $f(x) = x^2 - 3x + 2$

1st: Shape of the graph

-by looking at the value of a; $a = 1$

- the value of a>0, the graph will be MINIMUM graph.

2nd: Find y-intercept when $x = 0$

$$f(x) = 0 - 3(0) + 2$$

$$= 2$$

$$coordinate\ (0,2)$$

3rd : Find x intercept when $y = 0$

$$0 = x^2 - 3x + 2$$

$$x = 2; x = 1$$

$$coordinate\ (2,0); (1,0)$$

4th: Find the vertex ; Vertex (h, k)

- $h = \frac{-b}{2a}$
- $k = f(h)$

From the equation $f(x) = x^2 - 3x + 2$

$$a = 1; b = -3\ and\ c = 2$$

Substitute into $h = \frac{-b}{2a}$

$$h = \frac{-(-3)}{2(1)}$$

$$h = \frac{3}{2} = 1.5$$

to find k substitute h into $k = f(h)$;

$$k = f(h)$$

$$k = f(h^2 - 3h + 2)$$

$k = (1.5)^2 - 3(1.5) + 2)$

$k = -0.25$

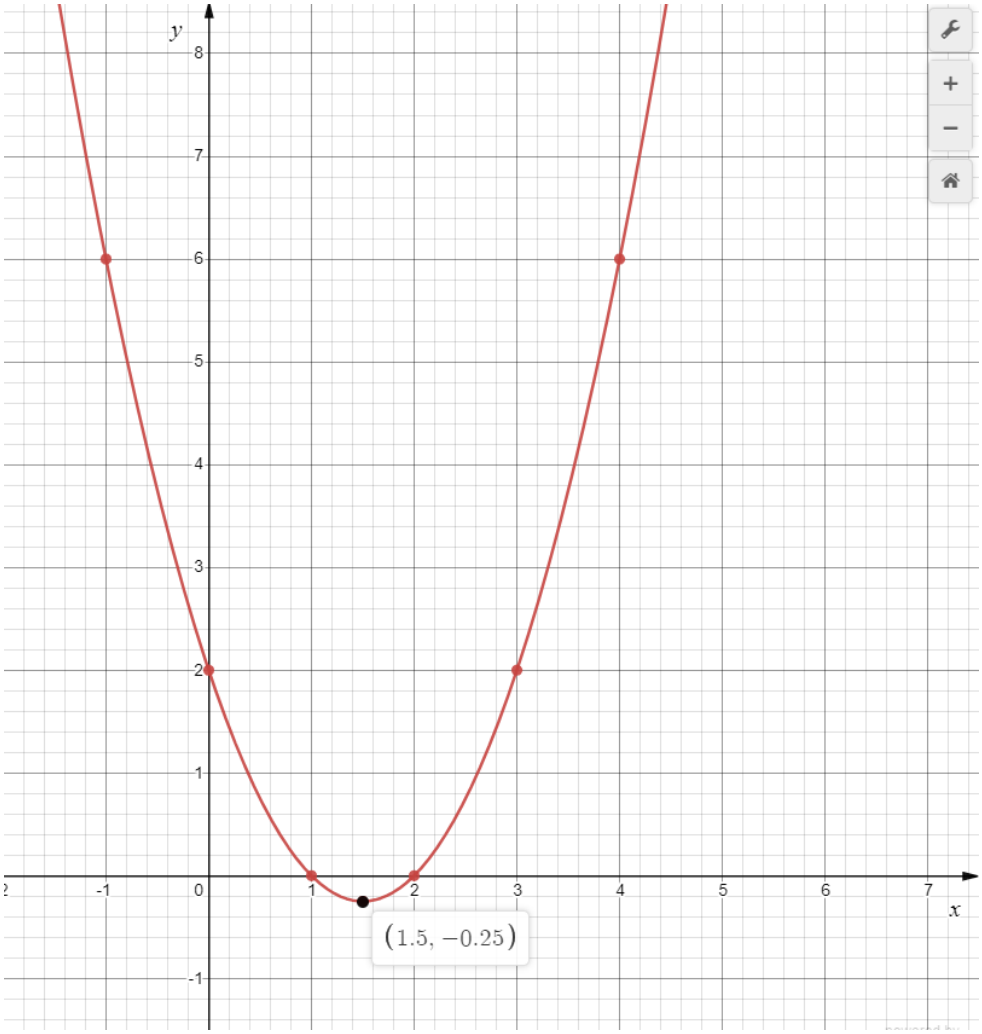
Thus vertex $(h, k) = (1.5, -0.25)$

5th : Build tables of values

VERTEX

x	-1	0	1	1.5	2	3	4
y	6	2	0	-0.25	0	2	6

6th: Sketch the graph.



EXAMPLE 2

Sketch a graph of $f(x) = -3x^2 + 27$

1st: Shape of the graph

-by looking at the value of a; $a = -3$

- the value of a<0, the graph will be MAXIMUM graph.

2nd: Find y-intercept when $x = 0$

$$f(x) = -3(0)^2 + 27$$

$$= 27$$

$$coordinate\ (0,27)$$

3rd : Find x intercept when $y = 0$

$$0 = -3x^2 + 27$$

$$x = -3; x = 3$$

$$coordinate\ (-3,0); (3,0)$$

4th: Find the vertex ; Vertex (h, k)

- $h = \frac{-b}{2a}$
- $k = f(h)$

From the equation $f(x) = -3x^2 + 27$

$a = -3; b = 0\ and\ c = 27$

Substitute into $h = \frac{-b}{2a}$

$h = \frac{-(0)}{2(-3)}$

$h = 0$

to find k substitute h into $k = f(h)$;

$k = f(h)$

$k = f(-3h^2 + 27)$

$k = -3(0)^2 + 27$

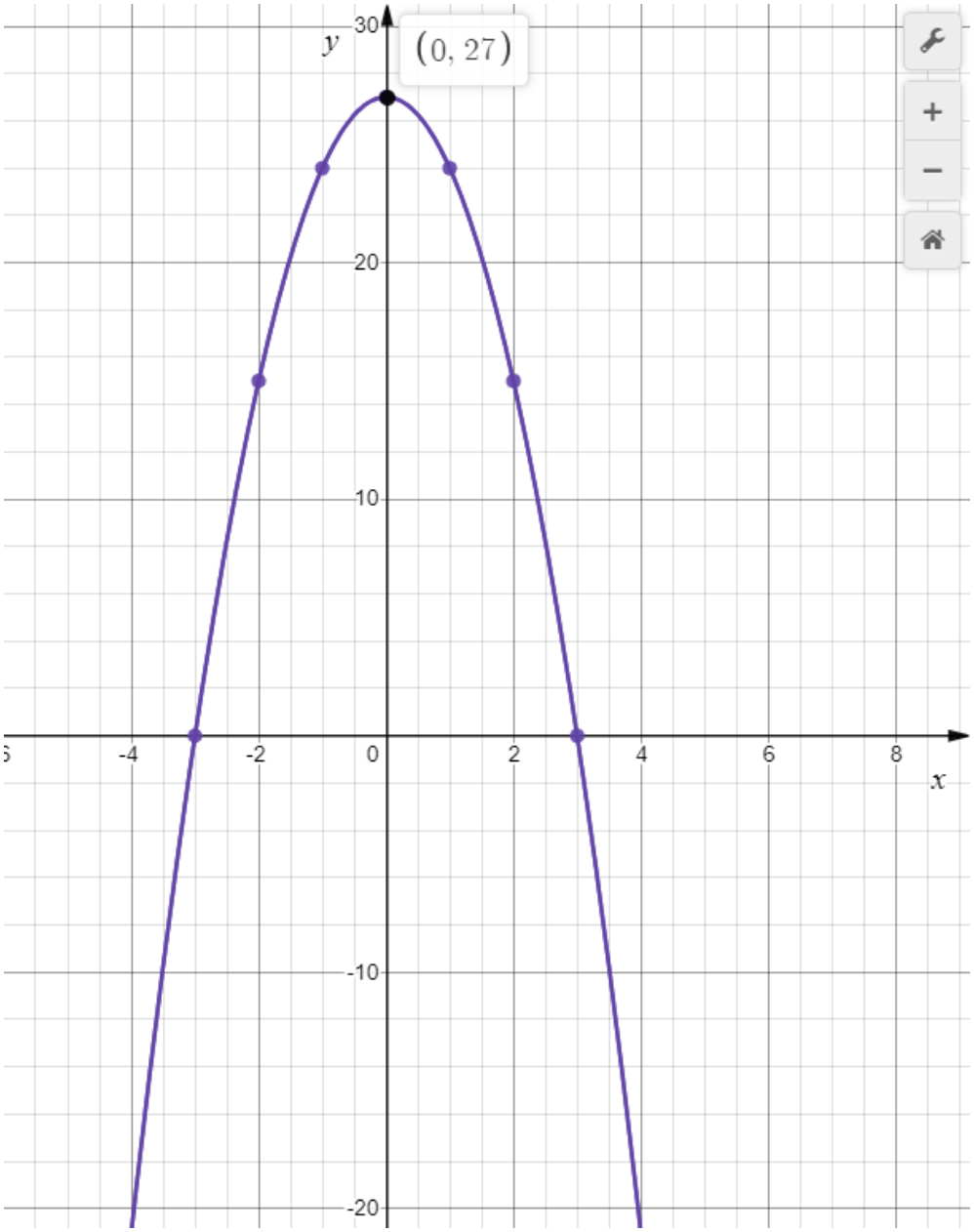
$k = 27$

Thus vertex $(h, k) = (0,27)$

5th : Build tables of values

VERTEX

x	-3	-2	-1	0	1	2	3
y	0	15	24	27	24	15	0



EXERCISE 4.2

1. Copy and complete the following table of values for the function given.

i. $y = 2x^2$

x	0	1	2	3	4
y			8	18	

ii. $y = x^2 - 2$

x	0	1	2	3	4
y	-2				14

2. Sketch a graph for the following function:

i. $y = x^2 + 1$

ii. $y = 6x^2 + x - 3$

iii. $y = 3x^2 + 2x$

iv. $y = -2x^2 + 8x + 4$

3. Draw the graph of $y = 2x^2 - 4x - 6$. Find the coordinate of x-intercept.

4. Complete table below for equation $y = 5 + 6x - 2x^2$

x	-1	0	1	2	3
y					

i. Using a scale of 2cm to 1 unit on x-axis and 2 cm to 2 units on y-axis, draw a graph of the function of $y = 5 + 6x - 2x^2$

ii. Based on the graph in (i), Find the vertex.

5. Draw the graph of $y = x^2$ for $-4 \leq x \leq 4$.

6. Draw the graph of $y = 4 - x^2$. From the graph, find the value of x if $y = 0$.

7. Draw the graph of $y = x^2 - 4x + 3$, then find the coordinates at x axis.

8. Given a function $y = 5x^2 - 9x - 5$

   i. Complete the table of values for the function above for $-2 \leq x \leq 3$

x	-2	-1	0	1	2	3
y	33		-5	-9		

   ii. Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, plot all the points.

   iii. Construct the graph of function.

   iv. From the graph, determine the value of x when y=0

9. Complete the table below for the function $y = x^2 - 2x - 3$, then sketch the graph.

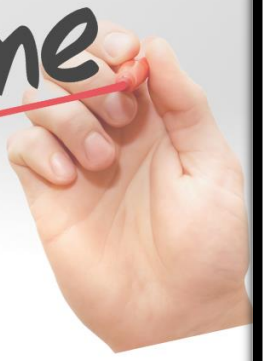
x	-2	-1	0	1	2	3	4
y	5		-3			0	5

10. Find the vertex for $y = 2x^2 - 1$

4.3: Simultaneous Equation

(Solve simultaneous equation using graphical method)

Learning Outcome



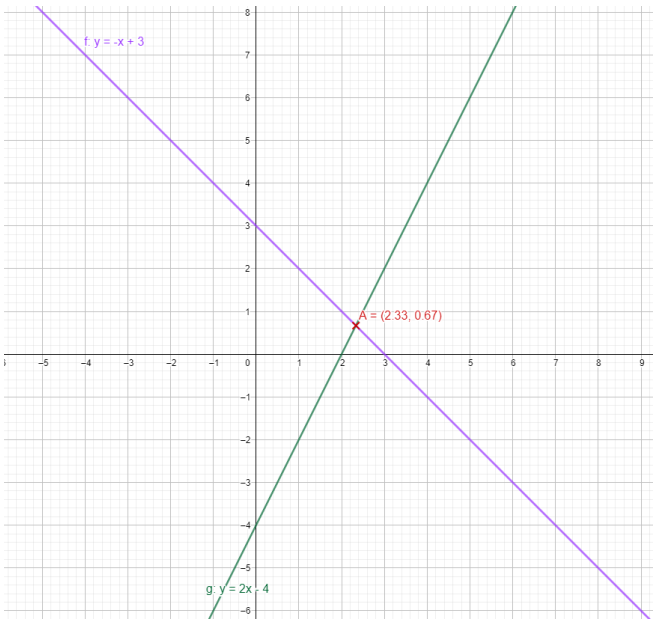
By the end of the lesson student should be able to:

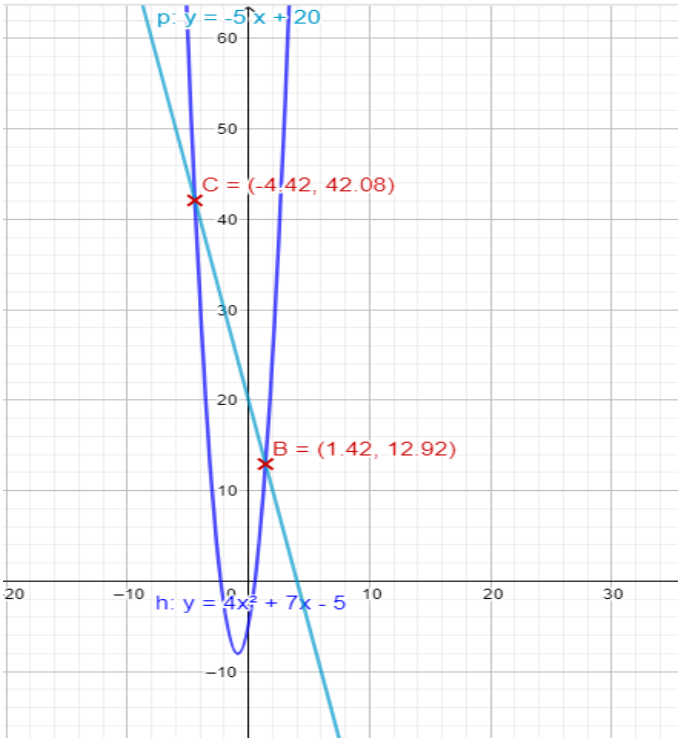
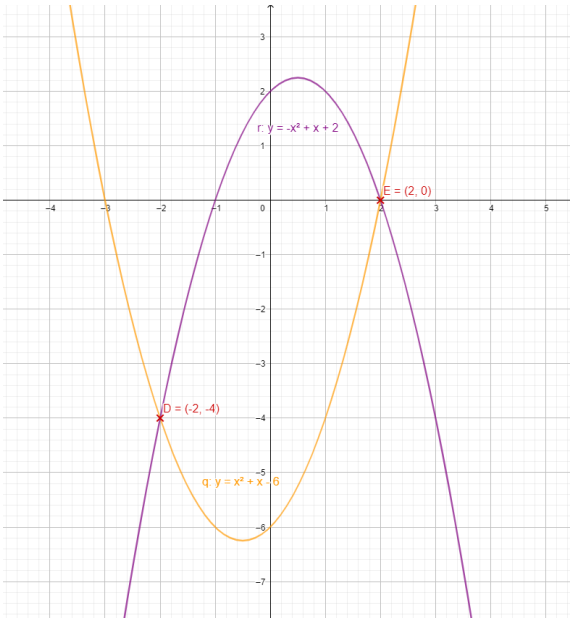
1. Find the point of intersection by solving the simultaneous equation by graphical method.

INTRODUCTION

We can solve an equation by finding the points of intersection of two appropriate graphs drawn on the same axes.

EXAMPLES

Graph	Example	Figure
Linear graph vs linear graph	Given a straight line for the function $y = -x + 3$ and it intersect to other straight line for the function $y = 2x - 4$. By using the graph, find the intersection point. From the graph, the intersection point is (2.33, 0.67).	

Graph	Example	Figure
Linear graph vs quadratic graph	Draw the graph of $y = 4x^2 + 7x - 5$ and $y = -5x + 20$ for $-6 \leq x \leq 4$ From the graph, determine the point of intersection. From the graph, coordinates point of intersection are (-4.42, 42.08) and (1.42, 12.92).	
Quadratic graph vs quadratic graph	Determine the point of intersection for the two curves with $y = x^2 + x - 6$ and $y = -x^2 + x + 2$ From the graph, the intersection points is (-2,-4) and (2,0).	

EXERCISE 4.3

1. Find the intersection point for the graph equation $y = x + 3$ and $y = -x + 6$.

   Ans : (1.5 , 4.5)

2. Find the intersection point for the graph equation $y = -3x + 4$ and $y = 3x - 2$

   Ans : (1 , 1)

3. The curve of $y = x^2 + 2x - 15$ and straight line of $y = -2x - 8$ are share the two of intersection points. Draw the suitable graph at the same axis and find the intersection points.

   Ans: (2.6, -2.8) and (-2.6, -13.2)

4. Find the intersection points from the two graphs of $y = x^2 - 9$ and $y = x + 2$.

   Ans: (-2.85, -0.85) and (3.85, 5.85)

5. Given the linear graph for equation $y = 4x + 8$ and quadratic graph for equation. $y = 16 - x^2$ meet twice at the same axis. Find the meet points from the graph drawing.

   Ans: (1.5, 14) and (-5.5, -14)

6. Given $y = -x^2 - 4x + 5$ and $y = -x + 5$ meet to intersection point. Draw the graph on the same axes, and then find the intersection points of the two graphs.

   Ans: (-3,8) and (0,5)

7. Given the equation $y = x^2 - 9$ and $y = -x^2 - 2x + 8$ which have two intersection points. Plot the graphs at the same axis and find the intersection points at the graph.

   Ans: (2.5, -3.3) and (-3.5, 2.8)

8. The curve line of $y = -x^2 - x + 2$ and $y = x^2 - x - 2$ meet twice at the same axis. Find the meet points from the graph drawing.

   Ans: (1.4, -1.4) and (-1.4, 1.4)

9. Given two quadratic equations $y = x^2 + 2x - 3$ and $y = -x^2 + 2x + 8$. Draw the graph on the same axes, and then determine the intersection points.

   Ans: (2.35,7.19) and (-2.35, -2.19)

10. Given $y = -2x^2 - 7x + 6$ and $y = x^2 + x - 2$ to be drawn on the same axes. Find the intersection points from the graph.

    Ans: (-3.44,6.4) and (0.77,-0.62)

REVISION QUESTION SET

REVISION SET 1

Q1. Copy and compute the table below for the function $y = 3x - 5$

x	-2	-1	0	1	2	3	4
y							

[4 marks]

Q2. Using a scale of 2cm to 2 units on the y-axis and 2 cm to 1 unit on the x-axis, draw the graph of $y = 3x - 5$

[6 marks]

Q3. By using the graph that has been drawn in 2, find:

i. y when $x = 1.2$ [2 marks]
ii. x when $y = 3.4$ [2 marks]

Q4. A straight line lies on $A(2,9)$ and $B(-4,-3)$. Find:

i. Slope, m [4 marks]

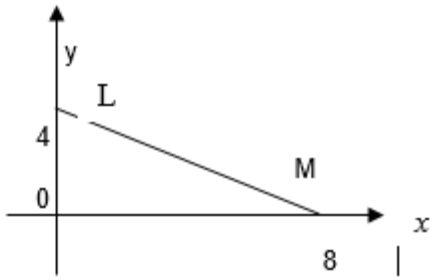
ii. y-intercept [3 marks]

iii. Distance of AB [4 marks]

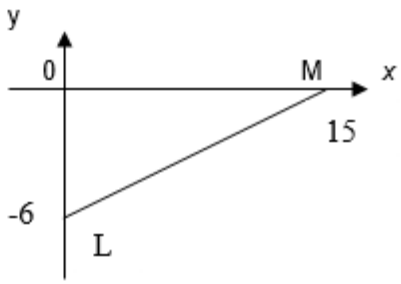
REVISION SET 2

Q1. Based on **Figure below** for each straight line in the following graph, find gradient and y-intercept. Then write the equation of the straight line.

i.



ii.



[10 marks]

Q2. Determine the intersection point for two straight line with equation $y = 3x$ and $y = -x + 4$ with graph plotting using the suitable scale.

[10 marks]

Q3. Given the midpoint of the points $(6, b)$ and $(-4,8)$ is $(1,10)$, find the value of b

[2 marks]

Q4. Given two points $(1, m)$ and $(n, -8)$. If the midpoint of AB is $(2, -2)$, find the values of m and n.

[3 marks]

REVISION SET 3

Q1: Sketch a graph of $2y + 3x = 6$ for $-2 \leq x \leq 2$

[5 marks]

Q2: Given a quadratic function $y = 2x^2 - 4x - 5$. Determine the:

[7 marks]

i. vertex point
ii. $x\ intercept\ \ and\ y\ intercept$
iii. Sketch the graph of $y = 2x^2 - 4x - 5$

Q3: Sketch the graph for $y = x^2 - 2x - 3$ and $y = 2x - 3$. Then find the intersection point.

[13 marks]

REVISION SET 4

Q1. Given point A$(1, -2)$) and point B(-3,5). Find

i. the gradient of AB [3 marks]

ii. the midpoint of AB [4 marks]

Q2. Table 1 shows the value of two variables x and y.

x	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7

Table 1

i. Based on the data given in Table 1,construct a graph y versus x [6 marks]

ii. From the graph in b(i), determine x-intercept and y-intercept. [2 marks]

Q3.

i. Construct a table of values for equation $y = 2x^2 - 8x + 6$ for $-0.5 \leq x \leq 4$ [5 marks]

ii. Draw the graph for equation in c(i) [5 marks]

REVISION SET 5

Q1. Given point U(-16,4) and point V(8,-20). Find

i. the gradient of UV [2 marks]

ii. the midpoint of UV [2 marks]

iii. the distance of UV [2 marks]

Q2. Find the intersection point between two lines for equation $y = x^2 - 2$ and $y = 2x + 1$ using graphical method for $-3 \leq x \leq 3$.

[10 marks]

Q3. Table 1 below shows the values of x and y of function $y = 2x^2 - 4x - 5$

x	-2	-1	0	1	2	3	4	5
y	11		-5	-7		1		25

Table 1

i. Complete the table above. [3 marks]

ii. Based on the answer in i , draw the graph of $y = 2x^2 - 4x - 5$ [6 marks]

REVISION SET 6

Q1. Sketch the graph for $y = 1 - 2x$ [5 marks]

Q2. Given a quadratic function $y = x^2 + 2x - 4$. Determine the [7 marks]

i. vertex point
ii. $x\ intercept\ \ and\ y\ intercept$
iii. Sketch the graph of $y = x^2 + 2x - 4$

Q3. Sketch the graph for $y = x^2 + x - 2$ and $y = -x + 5$. Then find the intersection point.

[13 marks]

REVISION SET 7

Q1. Given point A(-2,7) and point B(8,-2). Find the

i. gradient of line AB [2 marks]

ii. distance between point A and B [2 marks]

iii. y-intercept [2 marks]

Q2. Find the intersection point between two straight lines for equation $y = 2x - 7$ and $y = -x + 6$

[8 marks]

Q3. Table 1 below shows the values of x and y of function $y = -3x^2 - 5x + 16$

x	-4	-3	-2	-1	0	1	2	3
y	-12		14	18			-6	

Table 1

i. Complete the table above. [4 marks]

ii. Using the scale of 2 cm to represent 1 $unit$ on the $x - axis$ and a scale of 1 cm to represent 5 $unit$ on the $y - axis$, draw the graph of $y = -3x^2 - 5x + 16$ for $-4 \leq x \leq 3$

[7 marks]

REVISION SET 8

Q1. Based on the experiment of velocity versus time, the value of variable is recorded in Table 1 as shown below.

$Time\ , t(s)$	2	5	8	11	15
$Velocity,$ $v(m/s)$	16.9	19	21.1	23.2	26.0

Table 1

i. Plot a graph of velocity, $v\ versus\ time, t.$ [4 marks]

ii. Fine the velocity when $time = 10s$ [1 marks]

iii. Find value of time when $velocity = 20m/s$ [1 marks]

iv. Write the equation from the graph [4 marks]

Q2. Plot the graph showing $y = 2x^2 + 5x - 3$ and $y = -2x^2 - x + 6$ on the same graph paper by using your own scale.

[10 marks]

Q3. From graph in Q2, find the point of intersection of the graphs. [1 marks]

Q4. Sketch the graph for the following points.

i. A(0,5) and B(3,0) [2 marks]

ii. M(6,0) and N(0,-5) [2 marks]

REVISION SET 9

Q1. Given point A(4,10) and point B(-6,-4).

i. Plot point A and B in cartesian coordinate system [3 marks]

ii. Find the gradient of point A and B [3 marks]

Q2. The coordinates of the points P and M are $(5,3)$ and $(-2,4)$ respectively. If M is the midpoint of PQ, find

i. the coordinate of Q [3 marks]

ii. the distance between P and Q [3 marks]

iii. the slope of PQ [3 marks]

Q3. Consider $y = -2x + 1$

x	-1	1	1	2	3
y		1			

i. Complete the table above. [4 marks]

ii. Based on the table above, plot the graph of $y = -2x + 1$ [4 marks]

iii. From the graph in (ii), find the x-intercept and the y-intercept [2 marks]

REVISION SET 10

Q1. Sketch the graph for the following equations

i. $y = -x + 3$ [2 marks]

ii. $y = 2x$ [2 marks]

iii. $y = x^2 - 9$ [2 marks]

Q2.

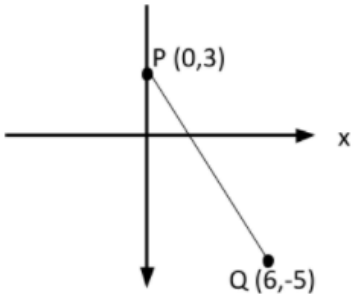


Diagram 1

Based on diagram 1,calculate

i. the gradient of PQ [2 marks]

ii. $x\ intercept\ \ and\ y\ intercept$ [4 marks]

iii. The midpoint of PQ [2 marks]

Q3. Show the intersection point of the given equations below by plotting a graph.

$$y = 2x + 4$$

$$y = -x + 5$$

[11 marks]

FUNCTION *And* GRAPH

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