POLYTECHNIC E-BOOK SERIES

NUMERICAL METHOD

ENGINEERING MATHEMATICS THREE

WORKBOOK

COMES WITH 100 ENRICHMENT QUESTIONS AND 6 SETS OF PAST FINAL EXAMINATION QUESTIONS IN RELATED TOPIC!

RAJA NADIA BINTI RAJA AHMAD
ROSLENA BINTI ABDUL RAHIM

"The Only Way to Learn Mathematics is to Do Mathematics"

~ Paul Halmos ~
Hungarian-American Mathematician

Department of Mathematics, Science and Computer
Polytechnic Nilai Negeri Sembilan
Kompleks Pendidikan Nilai
71760, Bandar Enstek
Negeri Sembilan Darul Khusus

ISBN

POLYTECHNIC E-BOOK SERIES

NUMERICAL METHOD

ENGINEERING MATHEMATICS THREE

WORKBOOK

COMES WITH 100 ENRICHMENT QUESTIONS AND 6 SETS OF PAST FINAL EXAMINATION QUESTIONS IN RELATED TOPIC!

RAJA NADIA BINTI RAJA AHMAD
ROSLENA BINTI ABDUL RAHIM

Numerical Method

ENGINEERING MATHEMATICS THREE

Workbook

FIRST ISSUE 2022

Publication@Polytechnic Nilai

WRITER

RAJA NADIA BINTI RAJA AHMAD
ROSLENA BINTI ABDUL RAHIM

EDITOR

ERNA RUHAILY BINTI RUSLI

A PUBLICATION BY:

POLITEKNIK MALAYSIA NILAI

KEMENTERIAN PENGAJIAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

e ISBN 978-967-2742-15-9

ENGINEERING MATHEMATICS 3: Numerical Method Workbook

A student workbook that contains a step-by-step answer guide as well as enrichment exercises related to specific topics and subtopics.

Department of Mathematics, Science and Computer
Polytechnic Nilai Negeri Sembilan
Kompleks Pendidikan Nilai
71760, Bandar Enstek
Negeri Sembilan Darul Khusus

COPYRIGHT DECLARATION

PUBLISHED BY

Polytechnic Nilai Negeri Sembilan
Ministry Of Higher Education Malaysia

ACKNOWLEDGEMENT

All praises to Almighty Allah,
The Most Merciful and
The Most Compassionate.
None of this would have been possible
without His blessings.
All of this started with a dream.
A dream to create lots of wonderful things
and success in many things.
We are so grateful to be a partner
together with dearest friends.

TABLE OF CONTENT

Topic/Subtopic	Page(s)

SYNOPSIS

This Numerical Method workbook provides students with guidelines and questions on solving linear and non-linear equations.

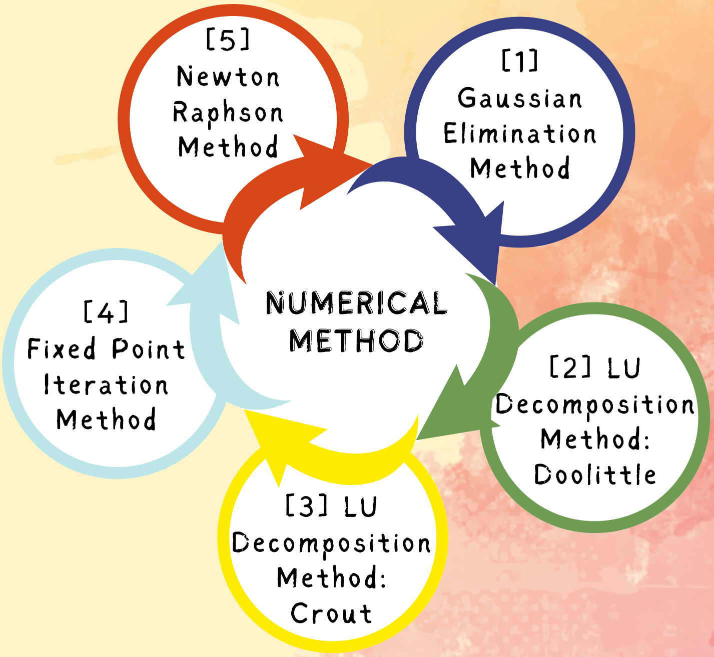
The methods used in this workbook to solve linear equations are the Gauss Elimination method and LU Decomposition method (Doolittle and Crout) while Simple Fixed Iteration and Newton Raphson methods are used for solving the non-linear equations.

A short and brief note as guidelines is included in this workbook. With these step-by-step guides, it would be a great help for the students to enhance their knowledge.

To master the mathematical concept of numerical methods, students can practice with the 100 questions in this workbook.

INTRODUCTION

The use of numerical methods allows for a better knowledge of phenomena and the exact prediction of anomalies that is not achievable using analytical approaches, which can only accurately answer problems involving two or three unknown variables. Numerical procedures are used when analytical techniques are unable or impractical to handle the mathematical issues involved in engineering analysis. Numerical methods are techniques for approximating mathematical procedures. Approximations are required because we cannot solve the procedure analytically or because the analytical method is intractable.



In this course, there are two numerical methods of solving the systems of equations are used:

DIRECT METHOD	ITERATIVE METHOD
[1] Gaussian Elimination, [2] LU Decomposition: Doolittle, [3] LU Decomposition: Crout.	[4] Fixed Point Iteration, [5] Newton Raphson.

Direct methods are more concise without the error of approximation obtained in a finite number of steps. However, iterative methods start with an approximate solution and then generate a sequence of solutions that modify the previous one to get an approximate answer.

GAUSSIAN ELIMINATION METHOD

The Gaussian Elimination method (also known as the Row Reduction Algorithm), is used to solve Linear Equations Systems problems.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

It comprises a set of operations performed on the related coefficient matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

To perform row reduction on a matrix, a set of elementary row operations must be carried out to transform the matrix into an Upper Triangular Matrix.

$$A' = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}' & a_{23}' \\ 0 & 0 & a_{33}" \end{bmatrix}$$

Sets of Elementary Row Operations (ERO):

Interchange between two rows	$R_1 \leftrightarrow R_3$
Multiply a row by a nonzero scalar	$R_2' \mapsto kR_2\,; k \neq 0$
Adding a row to another row	$R_3 \mapsto R_3 + kR_2$

GAUSSIAN ELIMINATION METHOD - STEP BY STEP

EXAMPLE 1: We can better understand this with the help of the example and Step-by-Step solution provided below.

Consider the system of equations	$2x+3y-z=5$ $4x+4y-3z=3$ $-2x+3y-z=1$
STEP 1 | Form an Augmented Matrix, (A | b)	$\left(\begin{array}{ccc\|c} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ -2 & 3 & -1 & 1 \end{array}\right)$
STEP 2 | Perform ERO on this matrix. (Allow any operation from sets of ERO, 1 or 2 or 3) **Goal: To convert the original matrix into an Upper Triangular Matrix**	$\left(\begin{array}{ccc\|c} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ -2 & 3 & -1 & 1 \end{array}\right) \begin{array}{l} R_2{}' \mapsto -2R_1+R_2 \\ R_3{}' \mapsto R_1+R_3 \end{array} \left(\begin{array}{ccc\|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 6 & -2 & 6 \end{array}\right)$ $\left(\begin{array}{ccc\|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 6 & -2 & 6 \end{array}\right) R_3{}'' \mapsto 3R_2{}'+R_3{}' \left(\begin{array}{ccc\|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & -5 & -15 \end{array}\right)$
STEP 3 | Convert the Upper Triangular Matrix formed into system of linear equation	$2x+3y-z=5$ $-2y-z=-7$ $-5z=-15$
STEP 4 | Solve the equation above using backward substitution	$-5z=-15 \longrightarrow z=3$ $-2y=-7+z \longrightarrow y=2$ $2x= 5-3y+z \longrightarrow x=1$
Therefore,	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

GAUSSIAN ELIMINATION METHOD - GUIDED EXERCISE

EXAMPLE 2: By using the Gaussian Elimination Method, fill in each blank box below with the correct answer.

Consider the system of equations

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x - y + 3z &= 9 \end{aligned}$$

STEP 1 | Form an Augmented Matrix, (A | b)

$$\left(\begin{array}{ccc|c} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array}\right)$$

STEP 2 | Perform ERO on this matrix. (Allow any operation from sets of ERO, 1 or 2 or 3)

Goal: To convert the original matrix into an Upper Triangular Matrix

$$\left(\begin{array}{ccc|c} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array}\right) \begin{array}{l} R_2' \mapsto \square \\ R_3' \mapsto \square \end{array} \left(\begin{array}{ccc|c} \square & \square & \square & \square \\ 0 & \square & \square & \square \\ 0 & \square & \square & \square \end{array}\right)$$

$$\left(\begin{array}{ccc|c} \square & \square & \square & \square \\ 0 & \square & \square & \square \\ 0 & \square & \square & \square \end{array}\right) R_3'' \mapsto \square \left(\begin{array}{ccc|c} \square & \square & \square & \square \\ 0 & \square & \square & \square \\ 0 & 0 & \square & \square \end{array}\right)$$

STEP 3 | Convert the Upper Triangular Matrix formed into system of linear equation

$$\begin{aligned} \square + \square + \square &= \square \\ \square + \square &= \square \\ \square &= \square \end{aligned}$$

STEP 4 | Solve the equation above using backward substitution

$$\begin{aligned} \square &= \square \rightarrow z = \square \\ \square &= \square \rightarrow y = \square \\ \square &= \square \rightarrow x = \square \end{aligned}$$

Therefore,

$$\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

GAUSSIAN ELIMINATION METHOD - INDEPENDENT PRACTICE

1- Write the given system of the linear equations below in the form of a matrix equation, Ax=b:

$$4x_1 + 2x_2 = 5 + x_3$$

$$x_1 + x_3 = 12 - 4x_2$$

$$4x_3 - x_2 - 12 = 2x_1$$

2- Write the given system of the linear equations below in the form of an augmented matrix [A|b]:

$$x_2 + 3x_1 - 5x_3 = P$$

$$Q = 2x_3 - 2x_2 + 4x_1$$

$$4x_3 - R - 2x_1 = x_2$$

$$\left(\begin{array}{ccc|c} 3 & 1 & -5 & P \\ 4 & -2 & 2 & Q \\ -2 & -1 & 4 & R \end{array}\right)$$

3- Consider the upper triangular matrix A is given by;

$$\begin{pmatrix} 2 & 3 & -1 \\ 0 & -2 & -1 \\ 0 & 0 & -5 \end{pmatrix}.$$

Find the value of $y_1, y_2,$ and y_3 when the value of b' is (5, -7, -15).

$y_1 = 1, y_2 = 2, y_3 = 3$

GAUSSIAN ELIMINATION METHOD - INDEPENDENT PRACTICE

3- Consider the given matrix A is given by;

$$\begin{pmatrix} 2 & 3 & -1 \\ 0 & -2 & -1 \\ 0 & 0 & -5 \end{pmatrix}.$$

Find the value of $y_1, y_2,$ and y_3 when the value of b' is (5, -7, -15).

$y_1 = 1, y_2 = 2, y_3 = 3$

4- Consider the given matrix A is given by;

$$\begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix}.$$

Find the upper triangular matrix A by performing elementary row operations.

$$A' = \begin{bmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{bmatrix}$$

5. Determine the value of a_{21}, a_{22}, a_{23} and b_2 by using Gaussian Elimination method.

$$-x - y - z = 26$$
$$4x - y - z = 16$$
$$x + 4y + z = 10$$

$a_{21} = 0, a_{22} = -5, a_{23} = -5,$
$b_2 = 120$

GAUSSIAN ELIMINATION METHOD - INDEPENDENT PRACTICE

6. Determine the value of a_{21}, a_{22}, a_{23} and b_2 by using Gaussian Elimination method.

$$-x - y - z = 26$$
$$4x - y - z = 16$$
$$x + 4y + z = 10$$

$a_{21} = 0, a_{22} = 2, a_{23} = -5,\ b_2 = -1$

7. Determine the value of a_{21}, a_{22}, a_{23} and b_2 by using Gaussian Elimination method.

$$y_1 + y_2 + 3y_3 = 8$$
$$2y_1 + 4y_2 + y_3 = 15$$
$$5y_1 + y_2 + 2y_3 = 19$$

$a_{21} = 0, a_{22} = 5, a_{23} = 4, b_2 = 0$

8. Determine the value of a_{21}, a_{22}, a_{23} and b_2 by using Gaussian Elimination method.

$$x + 4y + z = 7$$
$$-7x + y + 2z = -2$$
$$3y + 4z = 11$$

$a_{21} = 0, a_{22} = 29, a_{23} = 9, b_2 = 47$

9. Determine the value of a_{21}, a_{22}, a_{23} and b_2 by using Gaussian Elimination method.

$$x + 2y + 3z = 1$$
$$-2x + y - z = 4$$
$$-x + 3y + 2z = 5$$

$a_{21} = 0, a_{22} = 5, a_{23} = 5, b_2 = 6$

GAUSSIAN ELIMINATION METHOD - INDEPENDENT PRACTICE

10. Determine the value of a_{32}, a_{33} and b_3 by using Gaussian Elimination method.

$$-3y + 2z = -7$$
$$-10x + 3y + z = 9$$
$$x + 3z = 13$$

Answer: $a_{32} = 0, a_{33} = 3.3, b_3 = 13.2$

11. Determine the value of a_{32}, a_{33} and b_3 by using Gaussian Elimination method.

$$-x + 3y + z = 5$$
$$-y + 3z = 5$$
$$2x + y = 1$$

Answer: $a_{32} = 0, a_{33} = 23, b_3 = 46$

12. Determine the value of a_{32}, a_{33} and b_3 by using Gaussian Elimination method.

$$2x - z = 2$$
$$x + y - 3z = -2$$
$$-x - 3y + z = 6$$

Answer: $a_{32} = 0, a_{33} = 3, b_3 = 5.4$

GAUSSIAN ELIMINATION METHOD - INDEPENDENT PRACTICE

13. Determine the value of a_{32}, a_{33} and b_3 by using Gaussian Elimination method.

$$-x + 4y + z = 3$$
$$3x - 2y + 2z = -5$$
$$-4x + 2y - 6z = 1$$

Answer: $a_{32} = 0, a_{33} = -7, b_3 = -2$

14. Determine the value of a_{32}, a_{33} and b_3 by using Gaussian Elimination method.

$$-4x + 5y + z = -6$$
$$x + 2y + 3z = 2$$
$$3x - 5y + z = -4$$

Answer: $a_{32} = 0, a_{33} = 3, b_3 = 8.31$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

15. Use Gaussian Elimination method to solve each of the following system of linear equation.

a.

$$3x - y = 5$$
$$3x + z = 13$$
$$3x + y + z = 8$$

Answer: $x = 0, y = -5, z = 13$

b.

$$4x + 2y - 2z = 0$$
$$4x + y + z = 7$$
$$-4x + 5y + z = 3$$

Answer: $x = 0.9, y = 0.8, z = 2.6$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

c.

$$-x + y - 3z = -6$$
$$x + y + z = 9$$
$$x + y + 2z = 0$$

Answer: $x = 25.5, y = -7.5, z = -9$

d.

$$x + y + 4z = 11$$
$$2x + 7y + z = 7$$
$$4x + 3z = -2$$

Answer: $x = -2.84, y = 1.37, z = 3.12$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

e.

$$2x + 2y + 4z = 0$$
$$3x + y + z = 7$$
$$x + 3y + 4z = 5$$

Answer: $x = 1.5, y = 6.5, z = -4$

f.

$$12x + 6y = 16$$
$$2x + 3y + 2z = 10$$
$$6y + 13z = 32$$

Answer: $x = 0.21, y = 2.24, z = 1.43$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

g.

$$6x + 5y + 7x = 3$$
$$x - 4y + z = 6$$
$$3x + 2y = 5$$

Answer: $x = -1.37, y = 5.09, z = 28.09$

h.

$$3x - y - 4z = 18$$
$$2x + 5y + 2z = 14$$
$$4x + 2y - z = 12$$

Answer: $x = -2.67, y = 7.19, z = -8.3$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

i.

$$x + 2y + z = 7$$
$$3x + 3y - 4z = 16$$
$$2x + y + 3z = -1$$

Answer: $x = -0.92, y = 4.58, z = -1.25$

j.

$$2x - 3y + z = 9$$
$$-3x + 3y + z = 8$$
$$3x + 2y + z = 16$$

Answer: $x = 1.52, y = 1.10, z = 9.26$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

k.

$$x - 2y - 2z = 2$$
$$-2x - 2y + 3z = 0$$
$$x + 3y + 2z = 4$$

Answer: $x = 3.47, y = -0.95, z = 1.68$

l.

$$x + 2y + z = 2$$
$$3x + 6y + 2z = 1$$
$$2x - 4y + 3z = 1$$

Answer: $x = -5, y = 1, z = 5$

GAUSSIAN ELIMINATION METHOD - EXIT ACTIVITY

m.

$$2x + y + z = 0$$
$$3x + y = 1$$
$$3x + y - z = 1$$

Answer: $x = 1, y = -2, z = 0$

n.

$$4x + 4y + 2z = -18$$
$$x + 7y - 3z = -1$$
$$2x + 3y - z = -2$$

Answer: $x = -0.18, y = -2.06, z = -4.53$

LU DECOMPOSITION METHOD

Consider [A] is a square matrix of 3×3 ;

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

A matrix can be factored, that is, it can be expressed as the product of two matrices: [A] = [B][C]. There are numerous ways to obtain factor matrices. However, if we specify either the diagonal elements of [L] or [U], we will obtain a unique factorization for [A]. Doolittle and Crout's LU decomposition methods are based on these ideas. Similarly, [A] can be factored as the product of [L] and [U]; [A] = [L][U], where [L] is a lower triangular matrix and [U] is an upper triangular matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Doolittle's method yields a lower triangular matrix and a unit upper triangular matrix,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{l_{21}} & \mathbf{1} & \mathbf{0} \\ \boldsymbol{l_{31}} & \boldsymbol{l_{32}} & \mathbf{1} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

in contrast to Crout's method, which yields a unit lower triangular matrix and an upper triangular matrix,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \boldsymbol{u_{12}} & \boldsymbol{u_{13}} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{u_{23}} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

DOOLITTLE'S METHOD

Consider the system of linear equations,

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Then, by using LU Decomposition Doolittle's method, the system of linear equations can be solved in four steps:

STEP 1: Rewrite the linear equation into a matrix equation, AX = b

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

STEP 2: Construct the lower triangular matrix and upper triangular matrix, A = LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{l_{21}} & \mathbf{1} & \mathbf{0} \\ \boldsymbol{l_{31}} & \boldsymbol{l_{32}} & \mathbf{1} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

STEP 3: Solve Ly = b by using forward substitution

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

STEP 4: Solve UX = y by using backward substitution

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

DOOLITTLE'S METHOD - STEP BY STEP

EXAMPLE 1: Solve the given system of the linear equation below using Doolittle's method.

$$x + 2y + 3z = -4$$

$$2x + 6y - 3z = 33$$

$$4x - 2y + z = 3$$

STEP 1 | AX=b
Rewrite the linear equation into matrix equation

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & -3 \\ 4 & -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 33 \\ 3 \end{pmatrix}$$

STEP 2 | A=LU
Construct the lower triangular matrix and upper triangular matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & -3 \\ 4 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & -3 \\ 4 & -2 & 1 \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{pmatrix}$$

$u_{11} = 1$	$u_{12} = 2$	$u_{13} = 3$
$l_{21}u_{11} = 2$ $l_{21} = 2$	$l_{21}u_{12} + u_{22} = 6$ $u_{22} = 2$	$l_{21}u_{13} + u_{23} = -3$ $u_{23} = -9$
$l_{31}u_{11} = 4$ $l_{31} = 4$	$l_{31}u_{12} + l_{32}u_{22} = -2$ $l_{32} = -5$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 1$ $u_{33} = -56$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & -5 & 1 \end{pmatrix}; U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -9 \\ 0 & 0 & -56 \end{pmatrix}$$

STEP 3 | Ly=B

In this step, we use forward substitution method to determine the values of y

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & -5 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 33 \\ 3 \end{pmatrix}$$

$y_1 = -4$	$2y_1 + y_2 = 33$ $y_2 = 33 - 2(-4)$ $y_2 = 41$	$4y_1 - 5y_2 + y_3 = 3$ $y_3 = 3 - 4(-4) + 5(41)$ $y_3 = 224$

STEP 4 | UX=y

In this step, we use backward substitution method to solve the problem

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -9 \\ 0 & 0 & -56 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 41 \\ 224 \end{pmatrix}$$

$-56z = 224$ $z = -4$	$2y - 9z = 41$ $y = \frac{41 + 9(-4)}{2}$ $y = 2.5$	$x + 2y + 3z = -4$ $x = -4 - 2(2.5) - 3(-4)$ $x = 3$

Therefore,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \\ -4 \end{pmatrix}$$

DOOLITTLE'S METHOD - GUIDED EXERCISE

EXAMPLE 2: Solve the given system of the linear equation below using Doolittle's method.

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x - y + 3z = 9$$

STEP 1 | AX=b
Rewrite the linear equation into matrix equation

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

STEP 2 | A=LU
Construct the lower triangular matrix and upper triangular matrix

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

$$= \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

$$L = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}; U = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

DOOLITTLE'S METHOD - GUIDED EXERCISE

STEP 3 | Ly=B

In this step, we use forward substitution method to determine the values of y

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

STEP 4 | UX=y

In this step, we use backward substitution method to solve the problem

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

DOOLITTLE'S METHOD - INDEPENDENT PRACTICE

Generate the matrix A=LU such that L is the lower triangular matrix with principal diagonal elements being equal to 1 and U is the upper triangular matrix.

1)

$$\begin{aligned} x+2y+2z&=5\\ 3x+7y+5z&=20\\ 2x+7y+2z&=21 \end{aligned}$$

Answer: $L=\begin{bmatrix}1&0&0\\3&1&0\\2&3&1\end{bmatrix}, U=\begin{bmatrix}1&2&2\\0&1&-1\\0&0&1\end{bmatrix}$

2)

$$\begin{aligned} x+2y-5z&=8\\ x+2y-z&=4\\ -3x+4y+7z&=-1 \end{aligned}$$

Answer: $L=\begin{bmatrix}1&0&0\\-3&1&0\\1&0&1\end{bmatrix}, U=\begin{bmatrix}1&2&-5\\0&10&-8\\0&0&4\end{bmatrix}$

DOOLITTLE'S METHOD - INDEPENDENT PRACTICE

Generate the matrix A=LU such that L is the lower triangular matrix with principal diagonal elements being equal to 1 and U is the upper triangular matrix.

3)
$$-x + 4y + 5z = -5$$
$$-6x + y + z = -12$$
$$-x - y + 3z = 4$$

Answer: $L = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 1 & \frac{5}{23} & 1 \end{bmatrix}, U = \begin{bmatrix} -1 & 4 & 5 \\ 0 & -23 & -29 \\ 0 & 0 & \frac{99}{23} \end{bmatrix}$

4)
$$3x = -64$$
$$x - 2y + 4z = 8$$
$$x + 10y - 2z = 26$$

Answer: $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ \frac{1}{3} & -5 & 1 \end{bmatrix}, U = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 0 & 18 \end{bmatrix}$

DOOLITTLE'S METHOD - INDEPENDENT PRACTICE

Generate the matrix A=LU such that L is the lower triangular matrix with principal diagonal elements being equal to 1 and U is the upper triangular matrix.

5)

$$\begin{gathered} 2x + 3y + z = 2 \\ 7x + 7y + 2z = -1 \\ 2x + 5y - 2z = 7 \end{gathered}$$

Answer: $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{7}{2} & 1 & 0 \\ 1 & -\frac{4}{7} & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -\frac{7}{2} & -\frac{3}{2} \\ 0 & 0 & -\frac{27}{7} \end{bmatrix}$

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

6)

$$\begin{aligned} 2x - 2y + 3z &= 0 \\ 3x - y - 2z &= -7 \\ -x + 4y + 2z &= 2 \end{aligned}$$

Answer: x = -1.85, y = -0.43, z = 0.94

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

7)
$$3x - y + 3z = 2$$
$$5x + 3y - 2z = 1$$
$$5x - 2y + z = 3$$

Answer: x = 0.44, y = -0.33, z = 0.11

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

8)

$$x - y + 9z = 30$$
$$x + 4y + 3z = 2$$
$$5x + 2y + 3z = 3$$

Answer: x = -0.62, y = -1.75, z = 3.21

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

9)

$$3x + y + z = 10$$
$$-x + y + 3z = -7$$
$$2x - 2y - 2z = 0$$

Answer: x = 2.5, y = 6, z = -3.5

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

10)

$$5x + 4y + 9z = 60$$
$$3x + 4y + z = -3$$
$$-2x + 3y - z = -3$$

Answer: x = -3.42, y = -0.37, z = 8.73

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

11)

$$-7x - y + 2z = 3$$
$$y + z = -2$$
$$10x + y + 2z = 3$$

Answer: $x = 0.38, y = -3.22, z = 1.22$

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

12)

$$\begin{aligned} 5x + 8y - 4z &= 53 \\ 9x + 8y - 4z &= 93 \\ 7x + 2y - 3z &= 33 \end{aligned}$$

Answer: $x = 10, y = 9.81, z = 18.88$

DOOLITTLE'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Doolittle's method.

13)

$$9x + 7y + 5z = 35$$
$$2x + 8y + 2z = 39$$
$$-4x - 3y - 3z = 33$$

Answer: $x = 27.26, y = 13.2, z = -60.54$

CROUT'S METHOD

Consider the system of linear equations,

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Then, by using LU Decomposition Crout's method, the system of linear equations can be solved in four steps:

STEP 1: Rewrite the linear equation into a matrix equation, AX = b

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

STEP 2: Construct the lower triangular matrix and upper triangular matrix, A = LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \boldsymbol{u_{12}} & \boldsymbol{u_{13}} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{u_{23}} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

STEP 3: Solve Ly = b by using forward substitution

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

STEP 4: Solve UX = y by using backward substitution

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

CROUT'S METHOD - STEP BY STEP

EXAMPLE 1: Solve the given system of the linear equation below using Crout's method.

$$x + 2y + 3z = -4$$

$$2x + 6y - 3z = 33$$

$$4x - 2y + z = 3$$

STEP 1 | AX=b
Rewrite the linear equation into matrix equation

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & -3 \\ 4 & -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 33 \\ 3 \end{pmatrix}$$

STEP 2 | A=LU
Construct the lower triangular matrix and upper triangular matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & -3 \\ 4 & -2 & 1 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}\begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{pmatrix}$$

$l_{11} = 1$	$l_{11}u_{12} = 2$ $u_{12} = 2$	$l_{11}u_{13} = 3$ $u_{13} = 3$
$l_{21} = 2$	$l_{21}u_{12} + l_{22} = 6$ $l_{22} = 2$	$l_{21}u_{13} + l_{22}u_{23} = -3$ $u_{23} = -4.5$
$l_{31} = 4$	$l_{31}u_{12} + l_{32} = -2$ $l_{32} = -10$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 1$ $l_{33} = -56$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & -10 & -56 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -4.5 \\ 0 & 0 & 1 \end{pmatrix}$$

CROUT'S METHOD - STEP BY STEP

STEP 3 | Ly=B

In this step, we use forward substitution method to determine the values of y

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & -10 & -56 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 33 \\ 3 \end{pmatrix}$$

$y_1 = -4$	$2y_1 + 2y_2 = 33$ $y_2 = \frac{33 - 2(-4)}{2}$ $y_2 = 20.5$	$4y_1 - 10y_2 - 56y_3 = 3$ $y_3 = \frac{3 - 4(-4) + 10(20.5)}{-56}$ $y_3 = -4$

STEP 4 | UX=y

In this step, we use backward substitution method to solve the problem

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -4.5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 20.5 \\ -4 \end{pmatrix}$$

$z = -4$	$y - 4.5z = 20.5$ $y = 20.5 + 4.5(-4)$ $y = 2.5$	$x + 2y + 3z = -4$ $x = -4 - 2(2.5) - 3(-4)$ $x = 3$

Therefore,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \\ -4 \end{pmatrix}$$

CROUT'S METHOD - GUIDED EXERCISE

EXAMPLE 2: Solve the given system of the linear equation below using Doolittle's method.

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x - y + 3z = 9$$

STEP 1 | AX=b
Rewrite the linear equation into matrix equation

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

STEP 2 | A=LU
Construct the lower triangular matrix and upper triangular matrix

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

$$= \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

$$L = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} ; U = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

CROUT'S METHOD - GUIDED EXERCISE

STEP 3 | Ly=B

In this step, we use forward substitution method to determine the values of y

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

STEP 4 | UX=y

In this step, we use backward substitution method to solve the problem

$$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

CROUT'S METHOD - INDEPENDENT PRACTICE

Generate the matrix A=LU such that L is the lower triangular matrix and U is the upper triangular matrix with principal diagonal elements being equal to 1.

1)

$$-x + 2y + z = -8$$
$$2x - 5z = -14$$
$$4x + y + 3z = 16$$

Answer: $L = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 9 & \frac{55}{4} \end{bmatrix}, U = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix}$

2)

$$3x + 2y + z = 1$$
$$2x + 2y + z = 6$$
$$3x + y + z = 0$$

Answer: $L = \begin{bmatrix} 3 & 0 & 0 \\ 2 & \frac{2}{3} & 0 \\ 3 & -1 & \frac{1}{2} \end{bmatrix}, U = \begin{bmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

CROUT'S METHOD - INDEPENDENT PRACTICE

Generate the matrix A=LU such that L is the lower triangular matrix and U is the upper triangular matrix with principal diagonal elements being equal to 1.

3)

$$\begin{aligned} 2x + 5y + 2z &= 10 \\ x + 3y + 2z &= 7 \\ 2x + 7y + 7z &= 17 \end{aligned}$$

Answer: $L = \begin{bmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 2 & 2 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & \frac{5}{2} & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

4)

$$\begin{aligned} 2x + 5y + z &= 2 \\ 3x - y + 2z &= -4 \\ 2x + y + z &= 0 \end{aligned}$$

Answer: $L = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -\frac{17}{2} & 0 \\ 2 & -4 & -\frac{4}{17} \end{bmatrix}, U = \begin{bmatrix} 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{17} \\ 0 & 0 & 1 \end{bmatrix}$

CROUT'S METHOD - INDEPENDENT PRACTICE

Generate the matrix A=LU such that L is the lower triangular matrix and U is the upper triangular matrix with principal diagonal elements being equal to 1.

5)

$$\begin{aligned} 4x - y - z &= 18 \\ -x + 4y + z &= 18 \\ -x + 4y - z &= 4 \end{aligned}$$

Answer: $L = \begin{bmatrix} 4 & 0 & 0 \\ -1 & \frac{15}{4} & 0 \\ -1 & \frac{15}{4} & -2 \end{bmatrix}, U = \begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

6)

$$x + z = 20$$
$$-2x + 8y - 4z = 34$$
$$-3x + 6y + 9z = -8$$

Answer: $x = 4.45, y = 4.24, z = -2.24$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

7)

$$\begin{aligned} 5x + 8y + 9z &= -18 \\ -2x - 4y + 6z &= 43 \\ -3x + 8y + z &= 20 \end{aligned}$$

Answer: $x = -8.39, y = -1.1, z = 3.64$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

8)

$$3x - 3y + z = 8$$
$$2x + 3y + z = 9$$
$$3x + 2y + z = 16$$

Answer: $x = 8.6, y = 1.6, z = -13$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

9)

$$3x - 3y + 2z = 9$$
$$x + y + z = 10$$
$$2x + 3y - 3z = 8$$

Answer: $x = 4, y = 3, z = 3$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

10)

$$\begin{gathered}2x + 3y - 3z = 16\\ x - 4y + 9z = 20\\ 5x + 2y - 4z = 33\end{gathered}$$

Answer: $x = 7.57, y = 3, z = 2.71$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

11)

$$\begin{gathered} 9x + 2y + 4z = 53 \\ 8y + 6z = 34 \\ 3x - 3y + 2z = 13 \end{gathered}$$

Answer: $x = 4.06, y = 1.85, z = 3.2$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

12)

$$\begin{aligned} 2x - y + z &= 12 \\ 4x - y + 3z &= 9 \\ -2x + 5y + z &= -10 \end{aligned}$$

Answer: $x = 29.5, y = 16, z = -31$

CROUT'S METHOD - EXIT ACTIVITY

Solve the following system of linear equations using Crout's method.

13)

$$3x - 3y + 2z = 9$$
$$x + y + z = 10$$
$$2x + 3y - 3z = 8$$

Answer: $x = 4, y = 3, z = 3$

FIXED POINT ITERATION METHOD

The Fixed-Point Iteration approach turns algebraic and transcendental equations into fixed-point functions to iteratively identify the roots of those equations. A fixed point is one whose value remains constant after a specific transformation. A fixed point of a function in mathematics is a specific element that the function maps to itself. The Fixed-Point Iteration method computes the answer to the given problem by repeatedly applying the idea of a fixed point.

The ALGORITHM:

STEP 1
Find points a and b such that a<b

STEP 2
Take the interval [a,b] and find the average of a and b as the value of x_0

STEP 3
Formulate f(x)=0 in the form of x=g(x). List out all possible g(x).

STEP 4
Choose g(x) with minimum value of g'(x).
1-Differentiate all g(x) obtained with respect to x.
2-Substitute the value of x with value in STEP 2

STEP 5
Find the approximate root of f(x) by using g(x) that satisfy $|g'(x_0)| < 1$

FIXED POINT ITERATION METHOD - STEP BY STEP

EXAMPLE 1: Find the root of the function below by using the Fixed-Point Iteration method.

$$f(x) = x^3 - 3x - 5$$

Let f(x)=0

$$x^3 - 3x - 5 = 0$$

STEP 1 | Find point a and b such that a < b

x	0	1	2	3
$f(x)$	-5	-7	-3	13

STEP 2 | Find the average of a and b

$$x_0 = \frac{2+3}{2} = 2.5$$

STEP 3 | Formulate f(x)=0 in the form of x=g(x). List all possible g(x).

$$g_1(x) = \frac{x^3 - 5}{3} \qquad g_2(x) = \frac{5}{x^2 - 3} \qquad g_3(x) = \sqrt[3]{3x + 5}$$

STEP 4 | Choose g(x) which has the minimum value of g'(x).

1- Differentiate all g(x) with respect to x
2- Substitute the value of x with the initial value from STEP 2

$$g_1(x) = \frac{x^3 - 5}{3}$$

$$g_1'(x) = \frac{d}{dx}\left(\frac{x^3 - 5}{3}\right)$$

$$g_1'(x) = \frac{1}{3}(3x^2)$$

$$g_1'(x) = x^2$$

$$\therefore |g_1'(2.5)| = 6.25 > 1$$

$$g_2(x) = \frac{5}{x^2 - 3}$$

$$g_2'(x) = \frac{d}{dx}(5(x^2 - 3)^{-1})$$

$$g_2'(x) = 5(-1)(x^2 - 3)^{-2}(2x)$$

$$g_2'(x) = -\frac{10x}{(x^2 - 3)^2}$$

$$\therefore |g_2'(2.5)| = 2.37 > 1$$

$$g_3(x) = \sqrt[3]{3x + 5}$$

$$g'_3(x) = \frac{d}{dx}(3x + 5)^{\frac{1}{3}}$$

$$g'_3(x) = \frac{1}{3}(3x + 5)^{\frac{1}{3}-1}(3)$$

$$g'_3(x) = (3x + 5)^{-\frac{2}{3}}$$

$$\therefore |g'_3(2.5)| = \mathbf{0.19 < 1}$$

FIXED POINT ITERATION METHOD - STEP BY STEP

STEP 5 | Find the approximate root of f(x) by using g(x) that satisfy

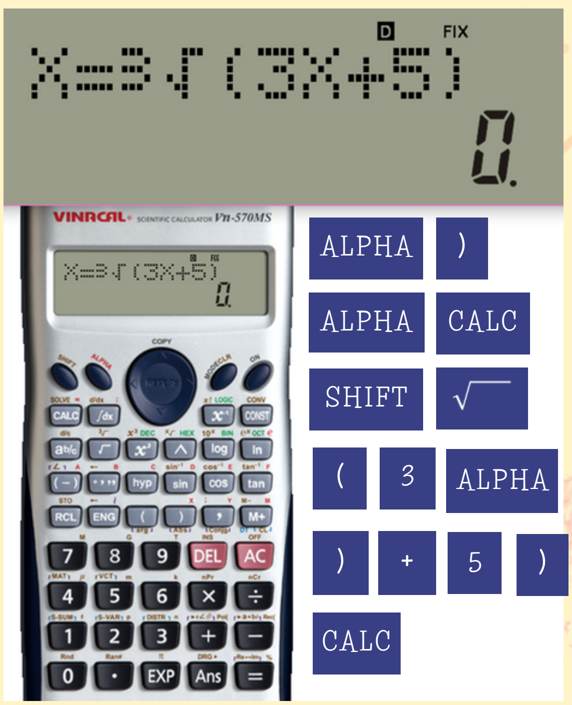
$$|g'(x_0)| < 1$$

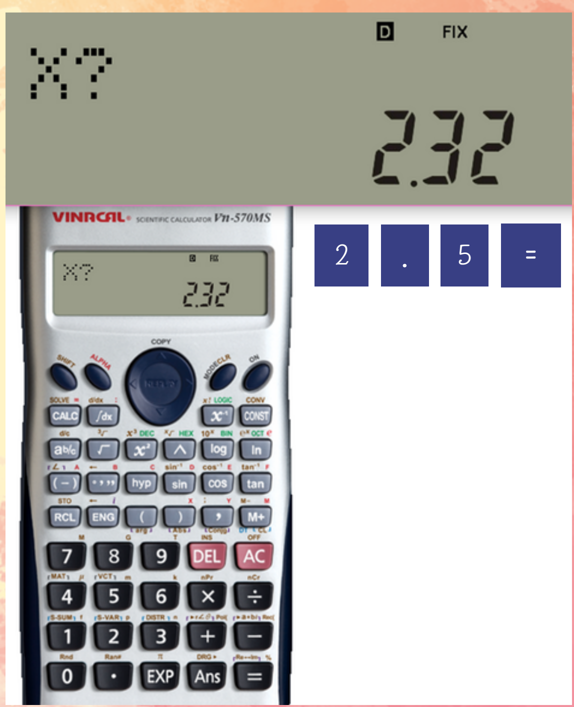
Use the CALC command on a scientific calculator, to calculate the value of g(x) by substituting the value of xo as the first iteration;

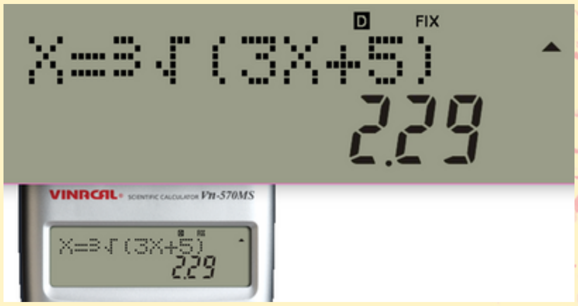
n	x_n	x_{n+1}	$\lvert x_n - x_{n+1}\rvert$
0	2.50	2.32	0.18
1	2.32	2.29	0.03
2	2.29	2.28	0.01

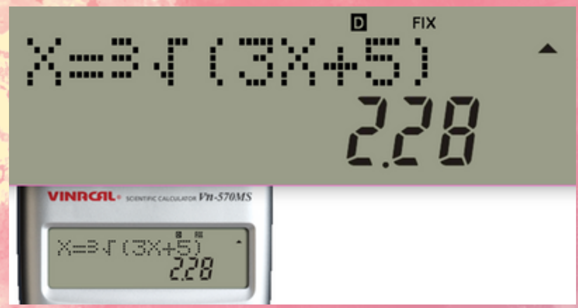
Therefore, the approximate root of $f(x) = x^3 - 3x - 5$ is at $x_2 \approx 2.28$

Calculation techniques using a scientific calculator









FIXED POINT ITERATION METHOD - GUIDED EXERCISE

EXAMPLE 2: Find the root of the function below using the Fixed-Point Iteration method.

$$f(x) = 2x^3 - 7x^2 - 6x + 1$$

STEP 1 | Let f(x)=0

STEP 2 | Find the initial value of x

x				
$f(x)$				

$$x_0 = \frac{\qquad}{2} =$$

STEP 3 | Formulate f(x)=0 in the form of x=g(x). List all possible g(x).

STEP 4 | Choose g(x) which has the minimum value of g'(x).

1- Differentiate all g(x) with respect to x
2- Substitute the value of x with the initial value from STEP 2

FIXED POINT ITERATION METHOD - GUIDED EXERCISE

STEP 5 | Find the approximate root of f(x) by using g(x) that satisfy $|g'(x_0)| < 1$

n	x_n	x_{n+1}	$\lvert x_n - x_{n+1}\rvert$
0			

FIXED POINT ITERATION METHOD - INDEPENDENT PRACTICE

List all possible g(x) for each of the non-linear equations below:

1) $x^3 - x + 1 = 0$

Answer: $x = x^3 - 1$, $x = \sqrt[3]{x+1}$

2) $x^3 + 4x = 1$

Answer: $x = \sqrt[3]{-4x+1}$, $x = \frac{-x^3+1}{4}$

FIXED POINT ITERATION METHOD - INDEPENDENT PRACTICE

List all possible g(x) for each of the non-linear equations below:

3) $3x^2 - x^3 + 3 = 0$

Answer: $x = \sqrt[3]{3x^2 + 3}$, $x = \sqrt{\frac{x^3-3}{3}}$

4) $e^x + x - 4 = 0$

Answer: $x = 4 - e^x$

FIXED POINT ITERATION METHOD - INDEPENDENT PRACTICE

5) $x^3 + \sin x = 1$

Answer: $x = \sqrt[3]{1 - \sin x}$

Formulate all possible functions of g(x). Then, determine the suitable function to iterate.

6) $f(x) = x^3 - 6x + 5$

Answer: $x = \sqrt[3]{8x - 5}$

FIXED POINT ITERATION METHOD - INDEPENDENT PRACTICE

Formulate all possible functions of g(x). Then, determine the suitable function to iterate.

7) $f(x) = 2x^4 - 8x - 2$

Answer: $x = \frac{x^4-1}{2}$

8) $f(x) = x^3 + 8x - 3$

Answer: $x = \sqrt[3]{3+8x}$

FIXED POINT ITERATION METHOD - INDEPENDENT PRACTICE

Formulate all possible functions of g(x). Then, determine the suitable function to iterate.

9) $f(x) = x^3 + 8x - 3$

Answer: $x = \sqrt[3]{3 + 8x}$

10) $f(x) = 6x^5 - x - 7$

Answer: $x = 6x^5 - 7$

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

1) **Find a root of an equation $f(x) = \sqrt{8 - x}$ using Fixed Point Iteration method. Give your answer correct to 3 decimal point.**
(Answer: x_6=2.372)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

2) **Find a root of an equation $f(x) = e^x - 3x^2 + 1$ initial solution x_0=1, using Fixed Point Iteration method. Give the answer correct to 3 decimal point.** (Answer: x_6=-2.947)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

3) **Find a root of an equation $f(x) = 1 + \frac{2}{x}$ initial solution x_0=2.2, using Fixed Point Iteration method. Give the answer correct to 3 decimal point.** (Answer: x_{12}=1.999)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

4) **Find a root of an equation $f(x) = x^2 + 5x - 7$ initial solution x_0=1.5, using Fixed Point Iteration method. Give the answer correct to 4 decimal point.** (Answer: x_4=1.1405)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

5) **Find a root of an equation $f(x) = x^3 - x + 1$ initial solution x_0=-1.5, using Fixed Point Iteration method. Give the answer correct to 4 decimal point.** (Answer: x_6=-1.3249)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

6) Find a root of an equation $f(x) = x - \tan(x)$ initial solution x_0=2, using Fixed Point Iteration method. Give the answer correct to 3 decimal point. (Answer: x_7=3.142)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

7) Find a root of an equation $f(x) = x^2 - 6x + 4$ initial solution x_0=0.5, using Fixed Point Iteration method. Give the answer correct to 4 decimal point. (Answer: x_7=0.7637)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

8) Find a root of an equation $f(x) = 3x^3 - 2x^2 - 4$ initial solution x_0=1.5, using Fixed Point Iteration method. Give the answer correct to 3 decimal point. (Answer: x_7=1.374)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

9) Find a root of an equation $f(x) = 6 - 4x^2$ initial solution x_o=1.5, using Fixed Point Iteration method. Give the answer correct to 4 decimal point. (Answer: x_2=1.2247)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

10) **Find a root of an equation $f(x) = e^{2x} - x^3 - 4$ initial solution x_0=2, using Fixed Point Iteration method. Give the answer correct to 2 decimal point.** (Answer: x_5=78.32)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

11) Find a root of an equation $f(x) = x^3 + 9x - 21$ initial solution x_0=3.5, using Fixed Point Iteration method. Give the answer correct to 4 decimal point. (Answer: x_6=3.8089)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

12) Find a root of an equation $f(x) = x^2 - 7x + 1$ initial solution x_o=0.5, using Fixed Point Iteration method. Give the answer correct to 3 decimal point. (Answer: x_3=0.146)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

13) Find a root of an equation $f(x) = 1.16x^2 - 6x + 1$ initial solution x_0=0.5, using Fixed Point Iteration method. Give the answer correct to 3 decimal point. (Answer: x_5= **0.173**)

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

14) Find a root of an equation $f(x) = 5x - 3x^2 + 3$ initial solution x_0=-0.5, using Fixed Point Iteration method. Give the answer correct to 3 decimal point. (Answer: x_{10}=- **0.468)**

FIXED POINT ITERATION METHOD - EXIT ACTIVITY

15) **Find a root of an equation $f(x) = x + 23 - x^3$ initial solution x_o=1.5, using Fixed Point Iteration method. Give the answer correct to 4 decimal point.** (Answer: x_3=1.3453)

NEWTON RAPHSON METHOD

The Newton-Raphson approach is a root-finding procedure used in numerical analysis that generates progressively improved approximations to a real-valued function's roots (or zeroes). The simplest form begins with a single-variable function f that is specified for a real variable x, the function's derivative f', and a first-guess value for the root of f, x_0. If the function is consistent enough and the initial estimation is accurate,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

is a more accurate approximation of the root than x_0. Until a result is obtained that is sufficiently accurate, the operation is repeated as,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

The ALGORITHM:

STEP 1: Find points a and b such that $a < b$

STEP 2: Take the interval [a,b] and find the value of x_0 using False Position Method

STEP 3: Evaluate the differential of $f(x)$

STEP 4: Draw an iterative table with 4 column: n, x_n, x_{n+1} and $|x_n - x_{n+1}|$

STEP 5: Iterates the value of x_i starting with x_0 obtained in STEP 2

STEP 6: Iteration stops when $|x_n - x_{n+1}| < 0.01$ or less (depends on number of decimal places required by the question.

NEWTON RAPHSON METHOD - STEP BY STEP

EXAMPLE 1: Find the real root of the function below using the Newton-Raphson method.

$$f(x) = x^3 + 2x^2 + x - 1$$

STEP 1 | Find point a and b such that a < b

x	0	1
$f(x)$	-1	3

STEP 2 | Find the initial value of x using False Position method

$$x_0 = \frac{1}{3-(-1)}\begin{vmatrix} 0 & -1 \\ 1 & 3 \end{vmatrix} = \frac{1}{4}(0-(-1)) = 0.25$$

STEP 3 | Find derivatives of f(x)

$$f'(x) = 3x^2 + 4x + 1$$

STEP 4 | Draw an iterative table with 4 column

STEP 5 | Iterates the value of x by using formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	x_{n+1}	$\lvert x_n - x_{n+1}\rvert$
0	0.250	0.529	0.279
1	0.529	0.469	0.060
2	0.469	0.466	0.003
3	0.466	0.466	0.000

STEP 6 | Iteration stops when $\lvert x_n - x_{n+1}\rvert < 0.001$

Therefore, the real root of the function

$$f(x) = x^3 + 2x^2 + x - 1$$

is at $x_3 \approx 0.466$

NEWTON RAPHSON METHOD - GUIDED EXERCISE

EXAMPLE 2: Find the real root of the function below using the Newton-Raphson method.

$$f(x) = x^3 + 5x - 42$$

STEP 1 | Find point a and b such that a < b

x				
$f(x)$				

STEP 2 | Find the initial value of x using False Position method

$x_0 =$

STEP 3 | Find derivatives of f(x)

$f'(x) =$

STEP 4 | Draw an iterative table with 4 column

STEP 5 | Iterates the value of x by using formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

STEP 6 | Iteration stops when $|x_n - x_{n+1}| < 0.001$

n	x_n	x_{n+1}	$\lvert x_n - x_{n+1} \rvert$
0			

Therefore,

NEWTON RAPHSON METHOD - INDEPENDENT PRACTICE

Determine the initial root, x_0 for each of the following equations.

1) $f(x) = 2\cos(x) - 6x + 4$

(Answer: x_0=0.5)

2) $f(x) = 3x^4 - 6x - 5$

(Answer: x_0=-0.5)

NEWTON RAPHSON METHOD - INDEPENDENT PRACTICE

Determine the initial root, x_0 for each of the following equations.

3) $f(x) = x - ln(x) - 3$

(Answer: x_0=4.5)

4) $f(x) = x^4 + 3x - 5$

(Answer: x_0=1.5)

NEWTON RAPHSON METHOD - INDEPENDENT PRACTICE

Determine the initial root, x_0 for each of the following equations.

5) $f(x) = 5x - 1 + e^x$

(Answer: x_0=0.5)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

1) Find the real root of the function $f(x) = \cos(x)$ with the initial solution x_o=2, using Newton Raphson method. Give the answer correct to 4 decimal point.

(Answer: x_3=1.5708)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

2) Find the real root of the function $f(x) = \cos(x) - x$ with the initial solution x_o=2, using Newton Raphson method. Give the answer correct to 4 decimal point.

(Answer: x_3=0.7391)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

3) Find the real root of the function $f(x) = x - \sqrt{(x+3)}$ with the initial solution x_o=1.5, using Newton Raphson method. Give the answer correct to 4 decimal point.

(Answer: x_2=**-3.0000)**

NEWTON RAPHSON METHOD - EXIT ACTIVITY

4) Find the real root of the function $f(x) = \frac{20}{x^2} - 7$ with the initial solution x_o=1, using Newton Raphson method. Give the answer correct to 4 decimal point.

(Answer: x_6=1.6903)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

5) Find the real root of the function n $f(x) = x^3 + 3x^2 - 5x + 4$ with the initial solution x_0=-1, using Newton Raphson method. Give the answer correct to 4 decimal point.

(Answer: x_8= **-2.0000)**

NEWTON RAPHSON METHOD - EXIT ACTIVITY

6) Find the real root of the function $f(x) = x^3 - \frac{7}{x} + 7$ with the initial solution x_o=1.5, using Newton Raphson method. Give the answer correct to 4 decimal point.

(Answer: x_3=1.9129)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

7) Find the real root of the function $f(x) = 2x^3 - \frac{10}{x^2}$ with the initial solution x_o=2, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_7=1.71)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

8) Find the real root of the function $f(x) = 3x^3 - 2x^2 - 4$ with the initial solution x_o=1.5, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_7=1.374)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

9) Find the real root of the function $f(x) = 2\sqrt{x} - \frac{15}{\sqrt{x}} - 10$ with the initial solution x_o=10.5, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_2=10.701)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

10) Find the real root of the function $f(x) = ln(x) + x - 5$ with the initial solution x_0=0.5, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_4=3.693

NEWTON RAPHSON METHOD - EXIT ACTIVITY

11) Find the real root of the function $f(x) = x^3 + 8x - 17$ with the initial solution x_0=1.5, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_3= **1.607**)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

12) Find the real root of the function $f(x) = 4sin\,(x) - \ln\,(x)$ with the initial solution x_o=3.5, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_3= **3.457**)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

13) Find the real root of the function $f(x) = 1 - e^x + 3\sin(2x)$ with the initial solution x_o=1.2, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: x_3=1.1609)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

14) Find the real root of the function $f(x) = x\sin(x) + 2x - 3$ with the initial solution x_o=1, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: x_3=0.5656)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

15) Find the real root of the function $f(x) = x + 13 - 10e^{0.5x}$ with the initial solution x_o=1.9, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: x_2=0.8394)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

16) Find the real root of the function $f(x) = 4x - 3 - e^{-x}$ with the initial solution x_o=0.5, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: x_4=0.8562)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

17) Find the real root of the function $f(x) = x^2 - e^{-x} - 2$ with the initial solution x_o=1.5, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: x_2=1.4917)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

18) Find the real root of the function $f(x) = 3x^3 - x - 6$ with the initial solution x_0=1.5, using Newton Raphson method. Give the answer correct to 3 decimal point.

(Answer: x_4=1.348)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

19) Find the real root of the function $f(x) = 2x^2 + x - 4$ with the initial solution x_0=1.5, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: x_4=1.1861)

NEWTON RAPHSON METHOD - EXIT ACTIVITY

20) Find the real root of the function $f(x) = \cos(2x) - 3x + 1$ with the initial solution $x_0=0.5$, using Newton Raphson method. Give the answer correct to **4** decimal point.

(Answer: $x_4=0.5086$)

COMPILATION OF PAST FINAL EXAMINATION QUESTIONS

NUMERICAL METHOD [QUESTION 2]

SET 1 - SESSION II: 2021/2022

QUESTION 2

A) i- Convert the following system of linear equations into $AX = B$ form:

(a) $9y - 6z = 5$
$7x + 9y - 2z = 6$ [2 marks]
$z + 8y = -3$

(b) $3p + 6q - 2r = 0$
$8p + 9q + 4 = 5r$ [2 marks]
$q + 3r = 3$

ii- Solve the following system of linear equations by using Gaussian Elimination Method.

$2x + y - 2z = 2$
$x + 2y = 3 - 2z$ [11 marks]
$3y + z = -1$

B) Given the equation; $x^4 - 2x^3 - x + 1 = 0$. Find the root of the equation by using Newton Raphson Method where the root is between $x = 0$ and $x = 1$. Give the answer correct to three decimal places. [10 marks]

QUESTION 2

A) Given a linear equation:

5r - 2s - 3t = -3
4s + 3t = -2
-s + 9t = 60

i- Rewrite the equation into the matrix form of Ax = B [1 mark]

ii- Solve r, s and t by using Crout's Method if given A = LU [9 marks]

$$A = LU$$

$$\begin{bmatrix} 5 & -2 & -3 \\ 0 & 4 & 3 \\ 1 & -1 & 9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 1 & -\frac{3}{5} & \frac{201}{20} \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix}$$

B) Given the non-linear equation is $5x^2 + 11x - 17 = 0$

i- Calculate the first approximate for x_0 [4 marks

ii- Calculate the root correct to 4 decimal places by using Newton Raphson's Method. [11 marks]

SET 3 - SESSION JUNE 2019

QUESTION 2

A) Solve the linear equations by using Gauss Elimination Method.

x + 2y - z = 2
3y + z + 4x = 3
2x + 2y + 3z = 5 [9 marks]

B) Based on the following equations:

a + 2b - 2c = 1
2a + 5b - 5c= - 2
- a + IOb - 5c = - 3

Calculate matrix L and U by using Doolittle Method. [10 marks

C) By using Newton-Raphson Method, determine the root for the given function below. Give the answer correct to three decimal places. Assume the first approximation as 1. [6 marks]

$$x^3 + 3x^2 - 2 = 0$$

SET 4 - SESSION JUNE 2019 (DBM3013)

QUESTION 2

A) Determine the roots for the function below, correct to 3 decimal places by using Fixed Point Iteration method. Given that $x_0 = 3$

$$x - e^{-x} = 0$$

[10 marks]

B) Solve the following equations by using the Gaussian Elimination Method.

3x - 6y + 5z = 6
- 4y + 3z = 4
4x + 8y - 8z = 10

[15 marks

SET 5 - SESSION DISEMBER 2018

QUESTION 2

A) By using Newton-Raphson method, determine the root for

$$5x^2 - 4x^{\frac{3}{2}} - 6 = 0.$$

Given $x_0 = 1.5$. Give the answer correct to four decimal places. [10 marks]

B) Find the matrix L and U for the equation below using Doolittle Method.

s + 4t - 2u = 3
3s - 2t + 5u = 14
2s + 3t + u = 11 [15 marks]

SET 6 - SESSION JUNE 2017

QUESTION 2

A) i- Convert the following system of linear equations into $AX = B$ form:

(a) $4y - 6z = 5$
$3x + 6y - 9z = -5$ [2 marks]
$-4x = 4$

(b) $2x + 6z + 2 = 0$
$x + 2y + 9z + 5 = 0$ [2 marks]
$6y - 6z = 5$

ii- Identify the real root by using the Newton-Raphson method correct to 3 decimal places for $f(x) = x^3 - x - 1$ where $x_0 = \sqrt{2}$. [6 marks]

B) Find the value $x_1, x_2, and\ x_3$ by using the Crout Method

$$2x_1 + x_2 + x_3 = 10$$
$$3x_1 + 2x_2 + 3x_3 = 18$$
$$x_1 + 4x_2 + 9x_3 = 16$$

[15 marks]

Numerical Methods Calculators - AtoZmath.com
https://atozmath.com
Online Calculator: Numerical Methods
https://www.codesansar.com/online-calculator
https://byjus.com/maths/newton-raphson-method

"The Only Way to Learn Mathematics is to Do Mathematics"

~ Paul Halmos ~
Hungarian-American Mathematician

Department of Mathematics,
Science and Computer
Polytechnic Nilai Negeri Sembilan
Kompleks Pendidikan Nilai
71760, Bandar Enstek
Negeri Sembilan Darul Khusus

e ISBN 978-967-2742-15-9

9 7 8 9 6 7 2 7 4 2 1 5 9